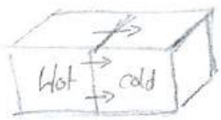
Thermal

1) Objects placed in thermal contact will experience a flow of thermal energy from $\underbrace{\underbrace{\text{hot object}}\ \text{to}\ \underbrace{\text{cold object}}}_{\text{determined by temprature}}$ until $\underbrace{\text{thermal equilibrium}}_{\text{objects have same Temperature}}$ is reached

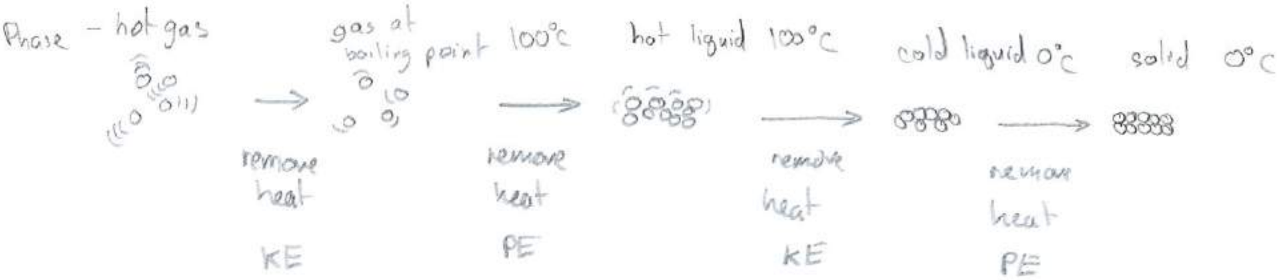


transfere of thermal energy

∴ Thermal energy = average KE of particles in the system → (changing thermal energy affects temp)

Heat = Transfer of Thermal energy (changing heat does not neceecarily change temp, but can affect state)
↳ from higher to lower temperatures

Internal Energy



- Temp. is related to average particle KE ($\frac{1}{2}m\langle v^2\rangle \propto T$

- 2 forms of energy ⇒ KE or potential
  - KE comes from movement of moleaules ($\underbrace{\text{translational KE}}_{\text{whole partid moving in a certain direction}}$ or $\underbrace{\text{rotational KE}}_{\text{particle rotating around a point}}$)
  - PE comes from intermolecular forces. Imagine pulling two molecules further apart, this requires work against PE

KE + PE = total energy = internal energy
↳ Internal energy is the sum of all individual KE & PE of particles in a body

Heat & Work

- Work is the energy transmitted from one system to another on macro level
- On micro level work is done we say heating has taken place. Either KE or PE increase

Heat Capacity

Heat capacity of a body is the amount of heat energy required to raise its temperature by 1 K Unit J/k Jk^{-1}

$$C = \frac{Q}{\Delta T}$$

Q ← Energy given to an object

ΔT ← change in temp

Heat capacity → C

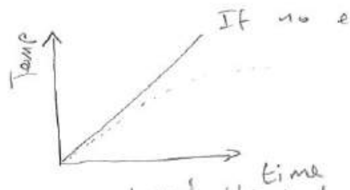
Specific Heat Capacity

Specific Heat capacity is energy required to raise 1 kg of material by 1 k. Unit J/kg k $Jkg^{-1}k^{-1}$

$$c = \frac{Q}{(m \Delta T)}$$

(small c) specific heat capacity → c

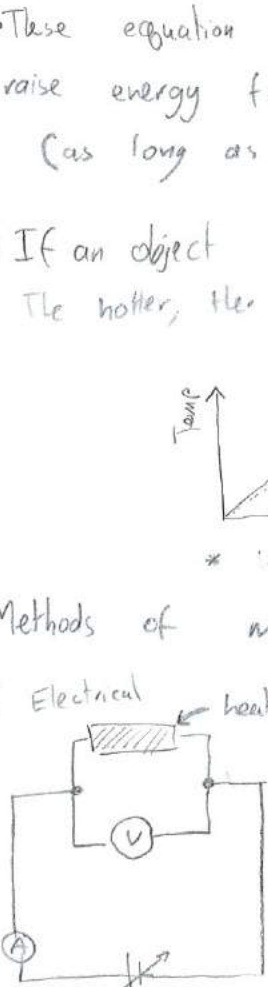
- These equation relate to thermal difference: That is same Q required to raise energy from 25°C to 50°C as from 500°C to 525°C (as long as no energy is lost from the object).
- If an object is raised above room temp it starts to lose energy. The hotter, the greater the rate at which it radiates heat



* object heated at constant rate.

Methods of measuring

1) Electrical



heater (placed in object.)

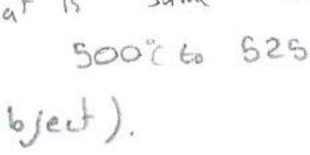
ΔT measured

variable PSU

$$c = \frac{ItV}{m(T_2 - T_1)}$$

experimental error
- loss of Q from apparatus
- container of substance also heated up
- takes time for heat to transfere

2) Mixture

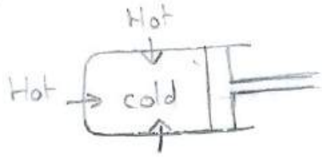


energy lost by hot substance = energy gained by cool substance

$$m_a c_a (T_a - T_{max}) = m_B c_B (T_{max} - T_B)$$

System concepts (Thermodynamics)

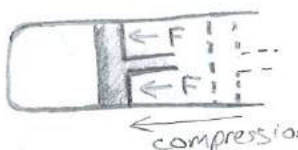
- Thermodynamic system → quantity of matter of fixed identity, around which we can draw a boundary. The boundries may be fixed or moveable and, work and heat may be exchanged through the boundry. Everything outside the boundry is surroundings
- Heat ΔQ = refers to the transfer of a quantity of thermal energy between the system and its surroundings. Transfer may be because of Temp difference



← arrows represent heat transfered.

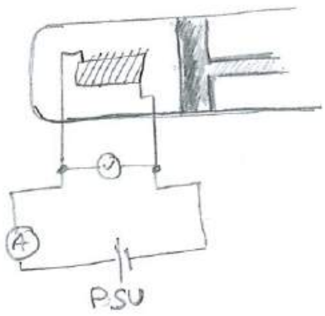
- Work ΔW = refers to macroscopic transfere of energy

1. work done = force * distance



when gas is compressed work is done on gas / When gas expands it does work on surrounding.

2 work done = $V \times t \times I$

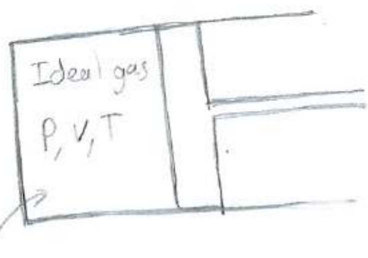


- Internal Energy U = only KE hence $\frac{1}{2} m_a \langle c^2 \rangle = \underbrace{\frac{3}{2} RT}$

ΔU = change in internal energy

internal energy of 1 mole of ideal gas.

A thermodynamic system



(U) internal energy

$U \propto T$

Surroundings

- Two ways to change internal energy of gas
  1. Adding Heat (Q)
  2. Moving piston - do work on gas or let it do work.

6) $P = \frac{N m \langle c^2 \rangle}{3V} = \frac{1}{3} \rho \langle v^2 \rangle$

Root mean: r.m.s

$\overrightarrow{1 m/s}$ $\overrightarrow{9 m/s}$ $\overrightarrow{4 m/s}$ $\overleftarrow{-2 m/s}$

$\overleftarrow{-8 m/s}$ $-4 m/s$

$\langle v \rangle = 0$

$\langle v^2 \rangle = \frac{1+64+81+16+16+4}{6} = 30.33$

$\sqrt{\langle v^2 \rangle} = \sqrt{30.33} = 5.5$

7) $PV = \frac{Nm}{3} \langle v^2 \rangle$

But $PV = nRT$ ← empical law

$nRT = \frac{Nm}{3} \langle v^2 \rangle$

8) if $n = 1$ $N \to N_A$ ← avogadro

$RT = \frac{N_a m}{3} \langle v^2 \rangle$

$\frac{3RT}{N_a} = m \langle v^2 \rangle$

$\frac{3RT}{2N_a} = \frac{1}{2} m \langle v^2 \rangle$

* $\frac{R}{N_a} = k = 1.38 \times 10^{-23}\ JK^{-1}$

② $\frac{3}{2} kT = \frac{1}{2} m \langle v^2 \rangle$

m ← mass of 1 particle

③ $\frac{3}{2} RT = \frac{1}{2} \underbrace{N_A m}_{\text{molar mass}} \langle v^2 \rangle$

$\frac{3}{2} RT = \frac{1}{2} M_m \langle v^2 \rangle$

① $PV = nRT$ ← temp

ideal gas constant: R; # of moles: n; Pressure: P; volume: V

Gas laws

$PV =$ constant — constant temp

$P \propto T$ — if T in k & constant volume

$V \propto T$ — if T in k & constant Pressure

$\left.\right\}$ real gasses follow these laws approximetly

$\frac{pV}{nT} =$ a universal constant $= R = 8.314\ J mol^{-1} k^{-1}$

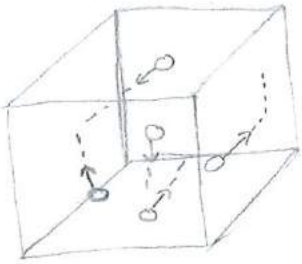
Ideal vs real gases

- Idea gasses follow the gas laws for all values of p, V, T, thus they cannot be liquefied.
- Real gasses approximetly behave as ideal gasses provided intermolecular forces are small enough to be ignored ⇒ pressure/density of gass must be low & temp high.

Kinetic model of an Ideal Gas

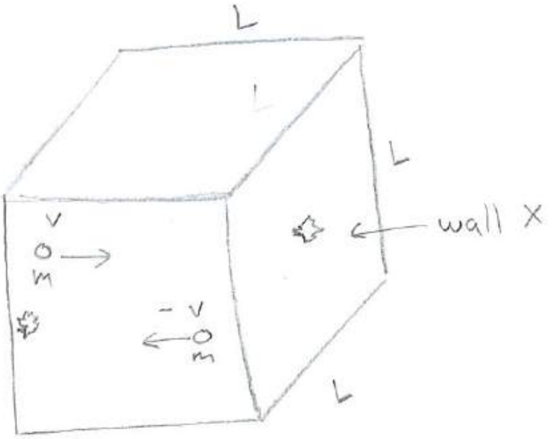
assumptions:

- Newtons laws apply
- there are no intra-molecular forces (no PE)
- molecules are treated as point masses
- collission between molecules are elastic
- collission are instantaneous
- Internal energy only dependent on temperature
- particle have random motion
- Particles have a range of velocities
- Large # of particles allows statistical methods to be used



pressure of gas is a result of collision between the molecules and the walls of the container.

Development of kinetic theory



1) collision on wall X causes Δp of $-2mv$
2) time between collisions $= \frac{2L}{V}$

Force exerted = $\underline{\text{rate}}$ of change of momentum

$$= \frac{-2mv}{2L/v} = \frac{-mv^2}{L} \Rightarrow \text{force on wall} = \frac{mv^2}{L}$$

3) pressure on wall $x = \frac{F}{A} = \frac{mv^2}{L} \times \underbrace{\frac{1}{L^2}}_{\text{Area}} = \frac{mv^2}{L^3} = \underbrace{\frac{mv^2}{V}}_{\text{volume}}$

4) $p = \frac{mv^2}{V}$ for N particles $P = N\frac{m\langle v^2\rangle}{V}$

mean of squares of velocity

5) only $\frac{1}{3}$ of particles moving in "x" direction:

$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ plane

$$P_x = \frac{\frac{Nm}{3}\langle v^2\rangle}{V} = \frac{Nm\langle v^2\rangle}{3V}$$

Specific latent heat

The specific latent heat of a material is the amount of heat energy required to change the phase of 1 kg of material at melting or Boiling point. Unit J/kg $J\,kg^{-1}$

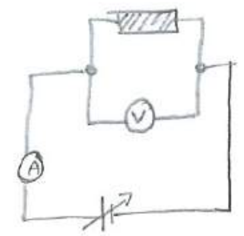
Fusion – liquid ⇌ solid
Vaporisation – liquid ⇌ gas

specific Latent heat $= L = \frac{Q}{m}$ $J\,kg^{-1}$

Measure of heat energy required to overcome PE ⇒ Breaking of intra molecular bonds requires energy

Measuring

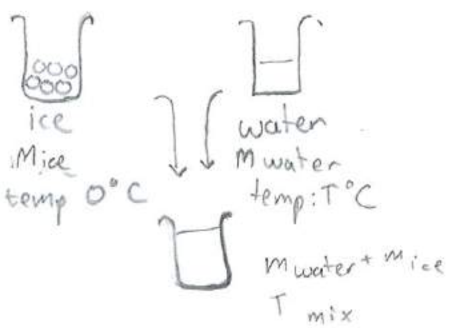
1) latent heat of vaporisation of water



$$L = \frac{ItV}{(m_1 - m_2)}$$

Experimental error
- Loss of thermal energy from apparatus
- some water vapour lost befor & after timing

2)



if no energy lost ⇒ energy lost by cooling water = energy gained by ice

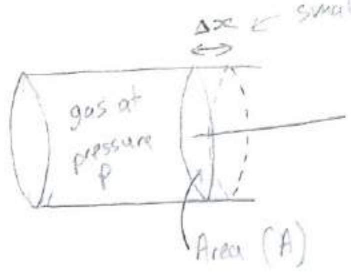
$$M_{water}\, c_{water}(T_{water} - T_{mix}) = \underbrace{m_{ice} L_{ice}}_{\text{latent}} + \underbrace{m_{ice}\, c_{water} T_{mix}}_{\text{heat capacity}}$$

Evaporation Vs Boiling

Evaporation occures when faster moving particles escape liquid (liquid particles have a range of speeds)
- only occurs on surface (occurs at any temp)
- increased temp ⇒ increased rate of evaporation
- The higher the pressure of air the loss evaporation
- Increased draught = increased evaporation (because ↑ is reduced)

Boiling occurs throughout the liquid when the average motion of particles is enough to over come forces holding them close together. This occurs at a specific temperature. (when saturated vapour pressure = atmospheric pressure)
- higher the pressure the higher the boiling point
- bubble form throughout liquid

Force & Work



$P = \frac{F}{A}$

$\therefore$ Force $= PA$

Work done by gas $= F \times d = PA\underbrace{\Delta x}_{\text{volume}} = P\Delta V$

at constant pressure work done $= P\Delta V$

1st law of thermodynamics:

the first law is a statement of conservation of energy. If amount of thermal energy ΔQ is given to a system, then one of two things must happen (or both) { system can increase its internal energy / system can do work

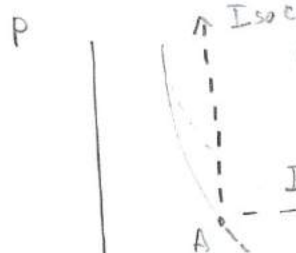
$$\Delta Q = \Delta U + \Delta W$$

ΔQ positive: thermal E going into system
negative: thermal E going out of system

ΔU positive: Internal energy is increasing ⇒ so is the temp
negative: Internal energy is decreasing ⇒ so is the temp

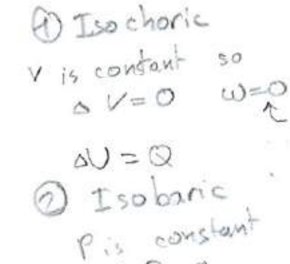
ΔW positive: system is doing work on surroundings (gas expanding)
negative: surroundings doing work on system (gas contracting)

PV & Work done diagram



$p\Delta V$ = work done in expanding from $A \Rightarrow B$

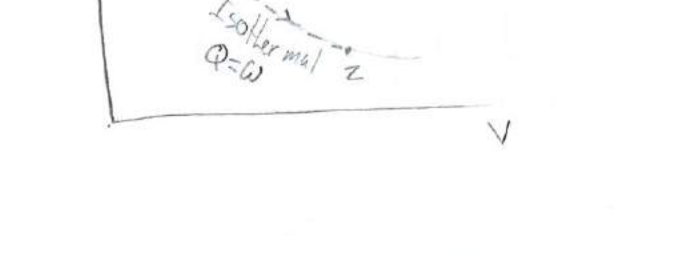
Isotherms



← isotherms (all points along the line have same temp)

further from origin the higher the temp

$T_2 > T_1$

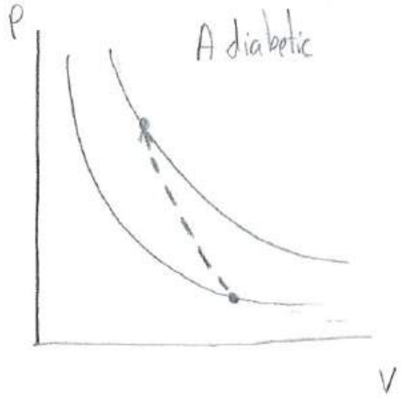


① Isochoric
V is constant so
$\Delta V = 0$ $\quad W = 0$ ← because $P\Delta V = W$

$\Delta U = Q$

② Isobaric
P is constant
$\Delta P = 0$
Expansion ⇒ ↑U ← because cutting through isotherms so ΔU +ve & W +ve = Q +ve
Contraction ⇒ ↓U so ΔU -ve and W -ve = Q -ve
heat lost

③ Isothermal
T constant
$\Delta U = 0$ $\quad (T \propto U)$
$Q = W$



- change where no heat is exchanged with surroundings

$Q = 0 \qquad \Delta U = -W$

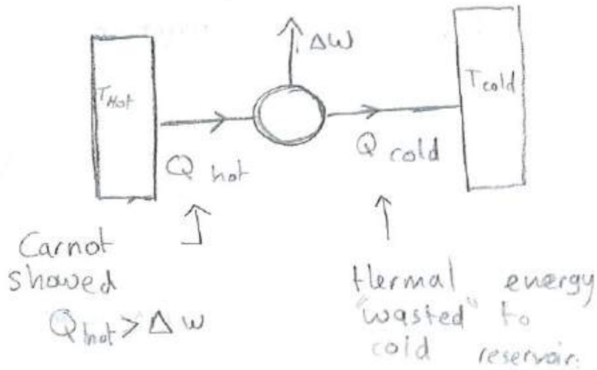
If work is done compressing system

$\Delta U\uparrow,\ T\uparrow,\ P\uparrow,\ V\downarrow$

- Adiabatic change must be fast so there is no time for heat to escape

Second law of thermodynamics & Entropy

- The conversion of thermal energy into work requires a cyclical process - a heat engine



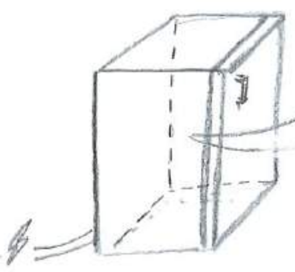
- This lead ⇒ No heat engine operating in a cycle, can take in heat from its surroundings and totally convert it into work
    - No heat pump can transfer thermal energy from low temp reservoir to a high temp reservoir without work being done on it.
        ⇒ heat flows hot → cold
- Entropy: Entropy of the universe can never decrease
    - Entropy expresses disorder in a system
      more heat recieved ⇒ more entropy the system has

$$\Delta S = \frac{\Delta Q}{T}$$

- Idea of degradatio ⇒ Energy that cannot be recovered.

Example

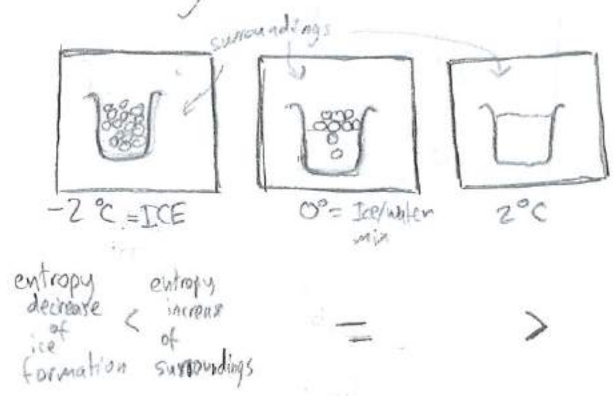
1) refrigirator (heat pump)



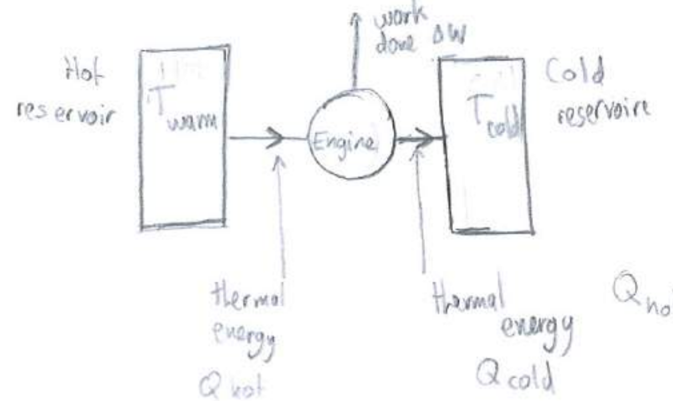
thermal energy taken out ice box & injected into surroundings

- Source of work is electricity

2) Water freezes as entropy increase in surrounding equals entropy decrease of water molecule becoming more ordered



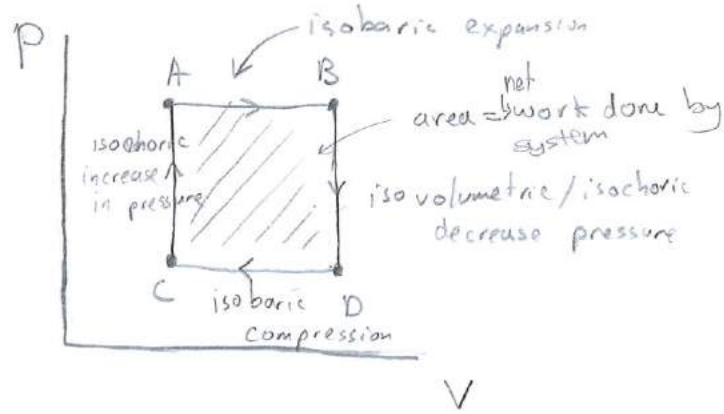
HEAT Engines



$Q_{hot} = W + Q_{cold}$

reservoir: implies constant temperature source or sink of thermal Energy. Taking or dumping thermal Energy can be done without changing temp of reservoirs.

Ideal gas in heat engines
- we can use a p-V diagram to represent 4 stage cycle.
- gas comes back to starting position but it has done work.
  - →To do this thermal energy must be taken from hot reservoir (during iso choric increase in pressure & isobaric expansion)
  - →different amount of thermal energy is injected back into the cold reservoire



$$\text{Efficiency} = \frac{\text{work done}}{\text{thermal energy taken from hot reservoir}}$$

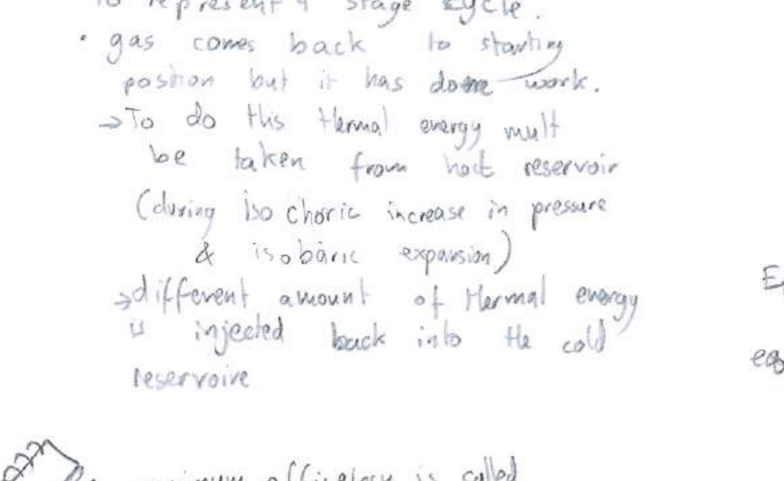
equivalent to

$$\text{Efficiency} = \frac{\text{rate of doing work}}{\text{thermal energy taken from hot reservoir}}$$

$$= \frac{\Delta W}{Q_{hot}}$$

maximum efficiency is called Carnot cycle.

Carnot Cycle & Carnot theorem



A→B = Isothermal expansion
B→C = Adiabatic expansion
C→D = Isothermal compression
D→A = Adiabatic compression

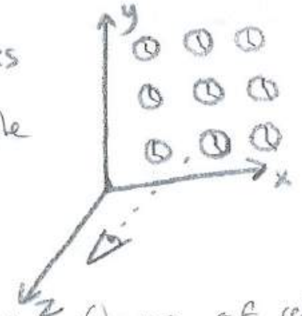
Temps of hot & cold res fix the maximum possible efficiency

$$e_c = 1 - \left(\frac{T_{cold}}{T_{hot}}\right) \quad (T \text{ in kelvin})$$

if operates at 300° C & ejects at 20°C $e_c =$

$$e_c = 1 - \frac{293}{573} = 0.49 = 49\%$$

Frames of reference

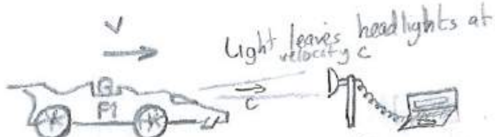


- definition: coordinate system covered in clocks than an observer used to measure the time and position of an event

– Inertial frame of reference (not accelerating): frame of reference where newtons law of motion apply

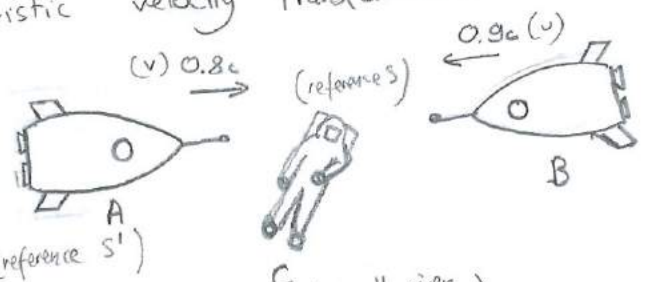
– Non-Inertial frame of reference (accelerating): a frame of reference that is accelerating relative to a inertial frame of reference. From within the frame seems a's Newtons laws do not apply.

Failure of Galilean transformation



light arrives to observer at c, (not v+c)

Relativistic velocity transformations



C (velocities all given relative to C)

velocity of B measured in S = u

velocity of A measured in S = v

If we want velocities relative to S' we use

$$U' = \frac{u-v}{1-\frac{uv}{c^2}}$$

(remember v = me, u = you, ∴ u_1 = your speed from me)

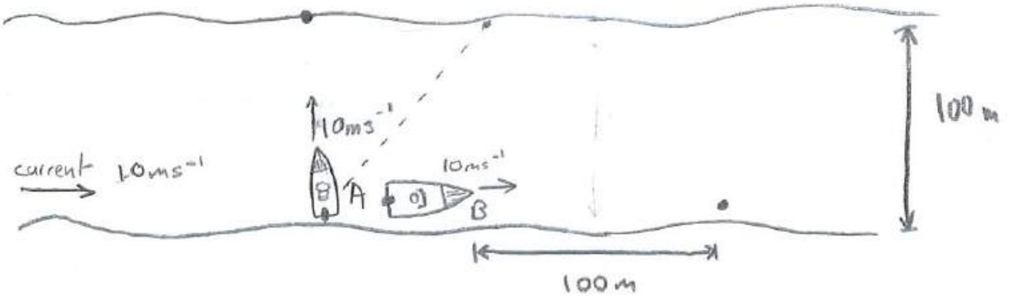
sub values in

$$u = \frac{0.9-0.8}{1-\frac{0.9 \cdot 0.8}{1}} = -0.988c$$

galilean transformation works on earth because velocities are not relativistic

Understanding light

- Imagine boat travelling in a river



This is an intro for what follows after !! ⇓

A

$\overset{\rightarrow}{10}$ $\uparrow 10$

it takes 10 sec to cross, so the current will act on boat for 10 sec, making go 100 m of coarse. final velocity = $10\sqrt{2}$

$t = 10$ sec

B

$\underset{\rightarrow\rightarrow}{10+10}$

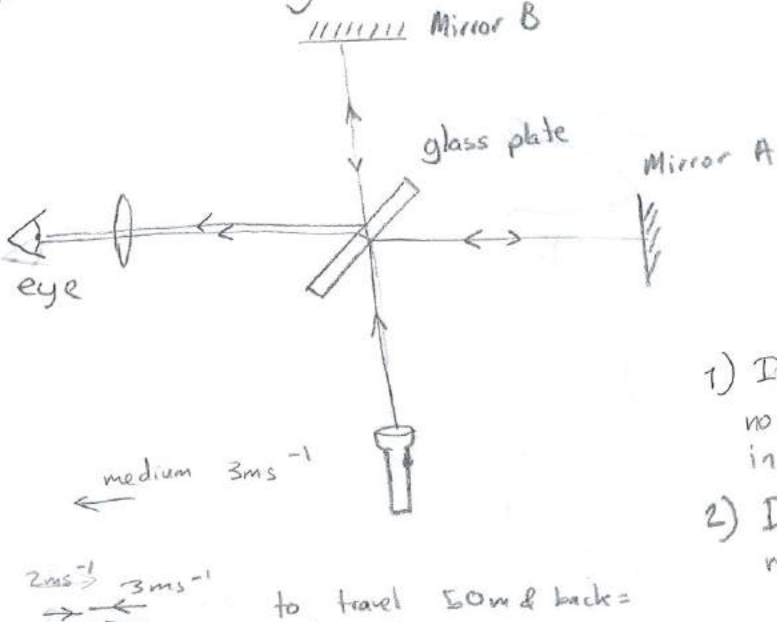
current acts in direction of boat so boat goes at $20\ ms^{-1}$

$t = 5$ sec

examples of how medium can affect speed

⚠ problems arise when it has to travel back!

- Michelson - Morley



When light hits glass plate some is reflected to A & some to B. Both rays over time will be reflected and interfere at the eye.

1) If both paths are equal no path difference, hence constructive interference.

2) If medium is flowing from right to left ← then A takes longer hence there is a path difference. eg. $\frac{\lambda}{2}$ causes deconstructive interference (& vice versa)

3) if medium flows at 45° angle to both A & B no path difference arises

eg

medium $3ms^{-1}$ ←

light parallel to medium: $2ms^{-1}$ →; $2ms^{-1}$ → ← $3ms^{-1}$; 5

perpendicular: $4ms^{-1}$ ↑; $3ms^{-1}$, $4ms^{-1}$, $5\ ms^{-1}$

to travel 50m & back = $\frac{50}{2} + \frac{50}{8} = 31.25$ sec

to travel 50m & Back $\frac{100}{4} = 25$ sec

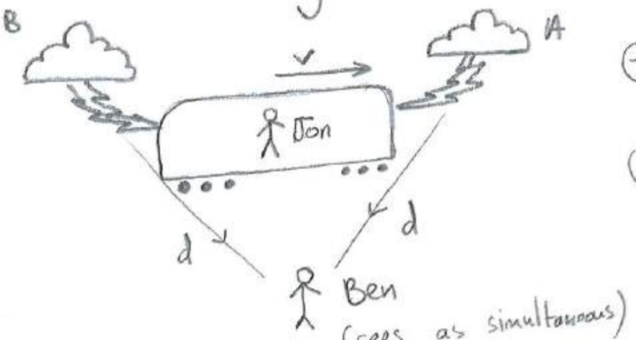
? they did not see a shift in the interference pattern irrispective of the rotation of the apparatus: light has no medium!

Einstien's Postulate

2 Speed of light in a vacuum is the same for all observers in all frame of reference.

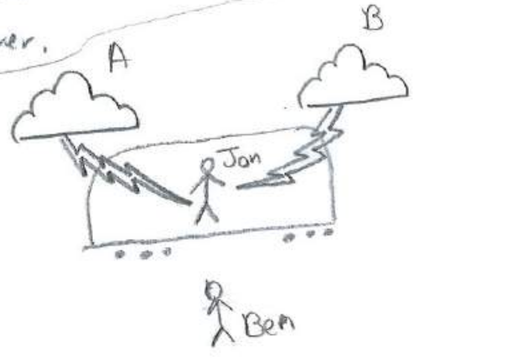
1 Laws of physics are the same in all inertial frames of reference.

Simultaneity



1. notice lighting are not in the same point in space
2. Jon moving at relativistic speed (v) towards A : light from A will travell less to reach Jon than light from B.
   ★ as speed of light is constant, Light will take less time to get to Jon from A. Hence A is before B.
3. Ben is equidistant (d) from A & B, and not moving so he observes A & B as symultaneous

⚠ symultaneity must be given at least relative to one observer.

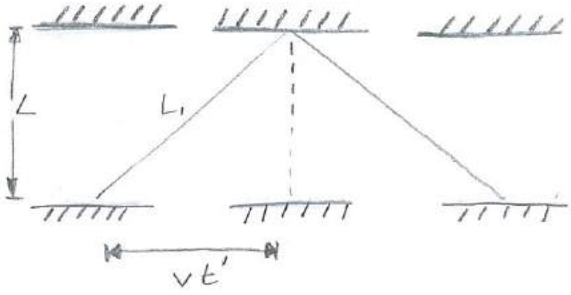


★ Same point in space, hence simultanious for both.

Events at 2 different points in space that are simultaneous in one frame of reference are not simultanious in all other frames of reference.

Time dilation:

Light clock



for viewer from different frame of reference path , therefore time seems to be going 'slower' in the clocks reference frame relative to external observer. (greater distance, same speed = more time)

$$L_1 = \sqrt{(vt')^2 + L^2}$$

$$t' = \frac{L_1}{c} = \frac{\sqrt{(vt')^2 + L^2}}{c}$$

$$t'^2 = \frac{v^2 t'^2 + L^2}{c^2}$$

$$t'^2\left(1 - \frac{v^2}{c^2}\right) = \frac{L^2}{c^2}$$ but $\frac{L^2}{c^2} = t^2$ (time as observed from within reference frame)

$$t'^2\left(1 - \frac{v^2}{c^2}\right) = t^2$$

$$t' = \underbrace{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}_{\text{lorentz factor } \gamma} \cdot t$$

$$t' = \gamma t_0$$

as seen from external observer

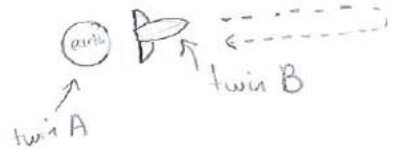
proper time

Proper time: time measured in a frame where event has taken place. (turns out to be the shortest time an observer could correctly record)

verification (Experimental)

• planes equiped with ceasium clocks.

Twin paradox

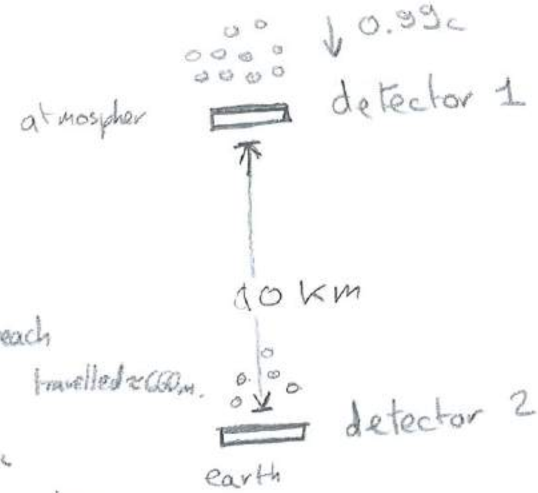


Twin A remains on earth while twin B travels to a distant star & back at relativistic velocity (hence time dilation occurs)

Paradox: Both twins think the other is moving relative to them, hence B think A should be younger & vice versa.

Solve: Twin B, to come back will have to $\underbrace{\text{decelerate \& re accelerate}}_{\text{non inertial}}$, which envolves external forces, hence situation is no longer symetrical. As A has not accelerated she must be correct

Length contraction



Muon decay

half life = 2 μs

Speed of muon = 0.99c $\therefore \gamma = 7$

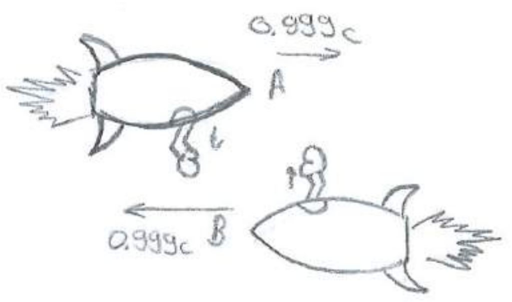
- without relativity no muon would be expected to reach earth, with only 2μs half life distance travelled ≈ 600m.
- however as muons are moving at relativistic speed from earth their half life appears longer as the muon time seems to be slowed down.
  - as seen from earth observer halflife = $2 \times 7 = 14$ μs
  - this allows many muons to reach earth as only 2.2 halflifes are experienced
- muons however have their clock running normally hence they measure the proper time of decay which is 2μs. To make results constant muons only have to travel $\frac{1}{7}$ of distance = 1.4 km ⇒ 2.2 halflifes.

$L_0 = \gamma L$

proper length

proper length: Length measured at rest relative to the object.

Relativistic Mass



① prepare to hit each other perpendicularly

② Each observes others punch to slower than their own due to time dilation

③ Each are suprised at the strength of the others punch considering the speed at which the others fist was moving

- only way that punch can feel stronger is if fist is heavier.

↳ $m = \gamma m_0$

relative mass ↑ — proper mass "rest mass"

Rest mass: mass of an object measured in a frame where the object is at rest.

Relativistic energy

KE = change in energy as it moves (energy due to movement, only consider the changed mass)

$KE = (m - m_0)c^2 = mc^2 - m_0c^2$
$= \gamma m_0 c^2 - m_0 c^2$
$=$ total E − rest E

→ $KE = \Delta E = (\Delta m)c^2$

As object goes faster $\gamma \uparrow \rightarrow m \uparrow, (\gamma m_0)$ hence KE↑

as velocity approaches c, γ tends to ∞ hence KE tends to infinity. This is why we cannot accelerate anything to c, as infinite energy would be required,

example

electron accelerated through p.d of 1 MeV

$KE = 1\,\text{Mev}$

$KE = (\gamma - 1) m_0 c^2$

$m_0 c^2 = 0.5$ (data booklet)

$1 = (\gamma - 1)\, 0.5$

$\therefore \gamma = 3$

$c\sqrt{1 - \frac{1}{(3)^2}} = v$

$= 0.94c$

$\begin{cases} \text{total } E = mc^2 = \gamma m_0 c^2 \\ KE = (\gamma - 1) m_0 c^2 \end{cases}$

Relativistic Momentum

$p = mv$ (non relativistic)

$p = \gamma m_0 v$ (relativistic equation)

Relation between Energy & momentum

$$E^2 = m_0^2 c^4 + p^2 c^2$$

E total energy (not rest E_0)

as photons $m_0 = 0 \Rightarrow E^2 = p^2 c^2 \Rightarrow E = pc$

we know (from Quantum & chem) $E = \frac{hc}{\lambda} = hf$

so $\frac{hc}{\lambda} = pc$

$\frac{h}{\lambda} = p$ ← back to debroglie

even if photon has no mass it has a momentum

Units

Energy MeV

Mass equivalent $MeVc^{-2}$

Momentum ⇒ $MeVc^{-1}$

↳ $E^2 = m_0^2 c^4 + p^2 c^2$

$(MeV)^2 = (MeVc^{-2})^2 c^4 + (MeVc^{-1})^2 c^2$

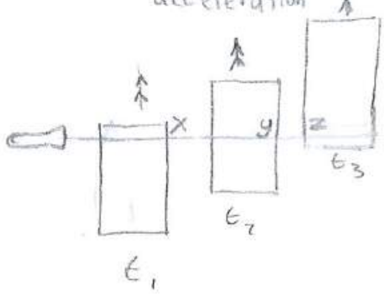
✓ balances

General Relativity

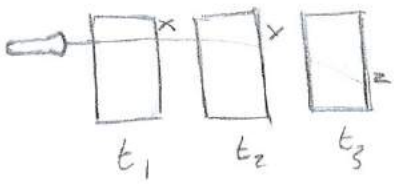
The Principle of equivalence

- No observer can determine by experiment whether they are in an accelerating frame of referance or a gravitational field

very large acceleration



As the box moves up light passes through it. From interior observer it looks like light is curving

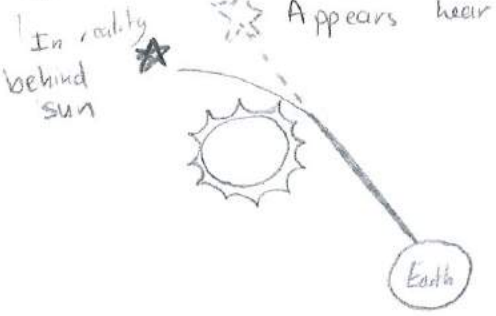


very large field

Gravity bends light ⇒ <u>but</u> light has no mass.
∴ think of gravity as a distortion in space time.

Gravitational mass $g = \frac{G m}{r^2}$ Inertial mass $F = ma$

Bending of light by the Sun

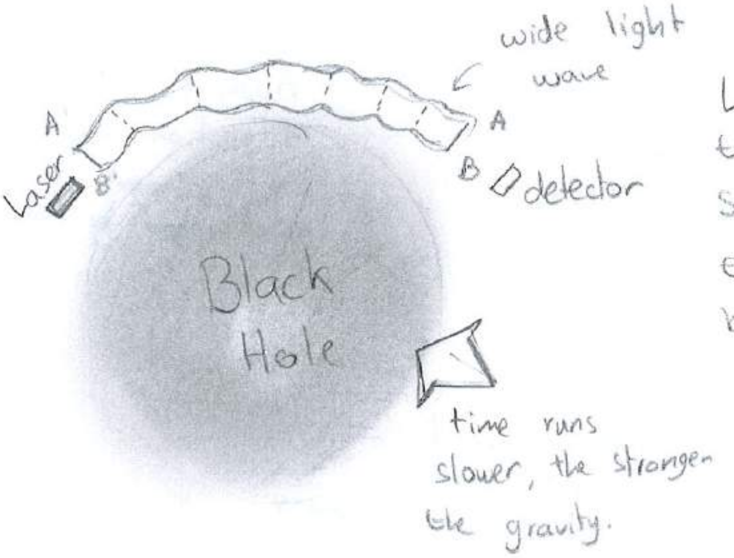


① Incoming light is bent by the sun (a large mass ⇒ distorts space time)

Bending of light (Gravitational lensing)

occurs if 2 galaxies (extremely large masses) are in line with observer.

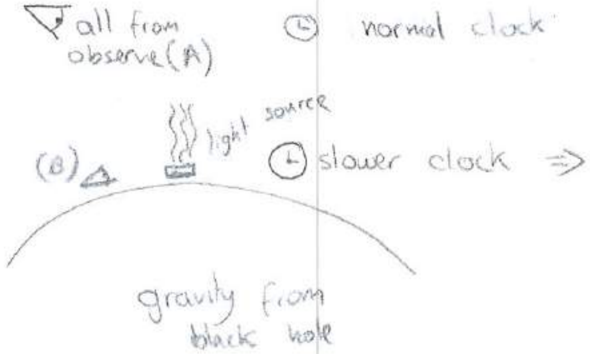
Slowing of Time By Large Masses



Light path $A \to A'$ arrives same time as light on path $B \to B'$. Since $B \to B' < A \to A'$ time elapsed on shorter path must be shorter.

Clocks near large masses tick slower

Gravitational red shift

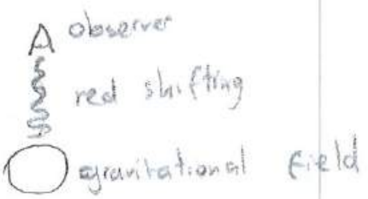


$\Rightarrow$ pendulum occillates slower, hence all occillations slower

$\therefore$ light emitted will appear to have lower f (Hz) hence larger λ. $\Rightarrow$ red shift

! yet observer B observes his clock as being correct, hence light is not red shifted

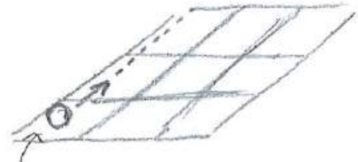
$\therefore$ red shift occurs on the way from source in gravitational field to the observer (A).



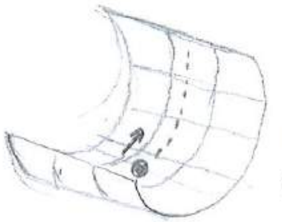
$E = hf$ energy lost due to field results in reduced f.

Curvature of space time

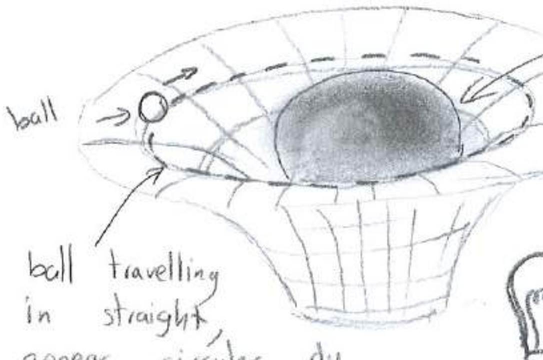
① consider 3D of space time in a 2d form



Applying newtonian laws of motion: object move in straight lines unless acted upon by external forces.



still straight path even though space time has been curved.

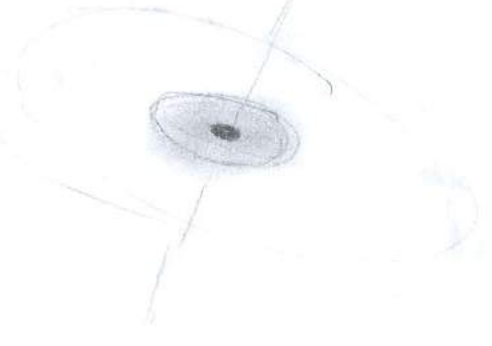


masses bend space time. the larger the mass the larger the dent

ball travelling in straight, appear circular du to bending of space time.

remember no forces needed as ball is travelling in straight line.

Black holes



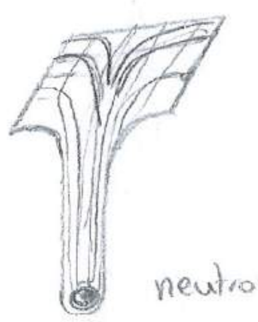
when mass is so large large space time is bend so much that light cannot escape.

$$R_S = \frac{2GM}{c^2} \leftarrow \text{mass}$$

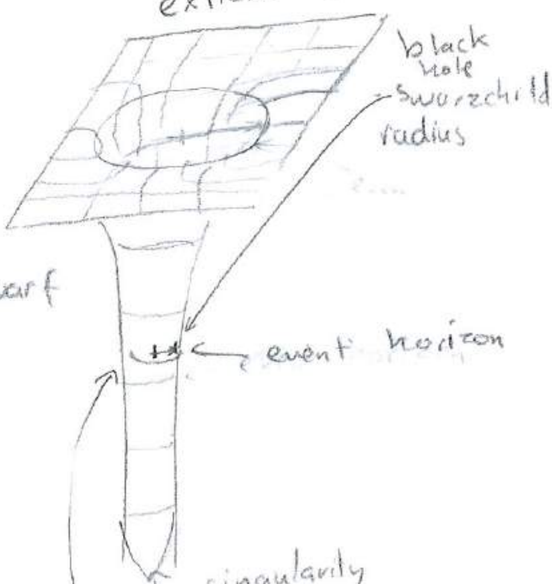
↑ swarzchild radius

time dilation: 

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{R_s}{r}}}$$



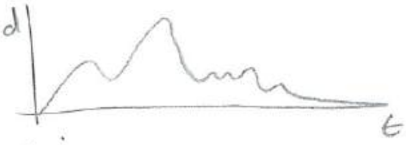
extreme case



to escape light needs more than the energy it has.

Digital technology

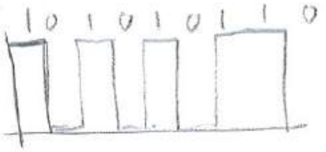
Analogue



analogue signals continuously vary with tim

- vinyl (LPs) - Cassette tapes
- Photo coping

digital



digital technique involve codes and signals that envolve large number of bits. (binary digits)

- PC - HDD - DVD

Binary

2^4 2^3 2^2 2^1 2^0 base 10

0 0 0 0 0 = 0

1 1 1 1 1 = 16+8+4+2+1 = 31

ASCII

where n = number of bits

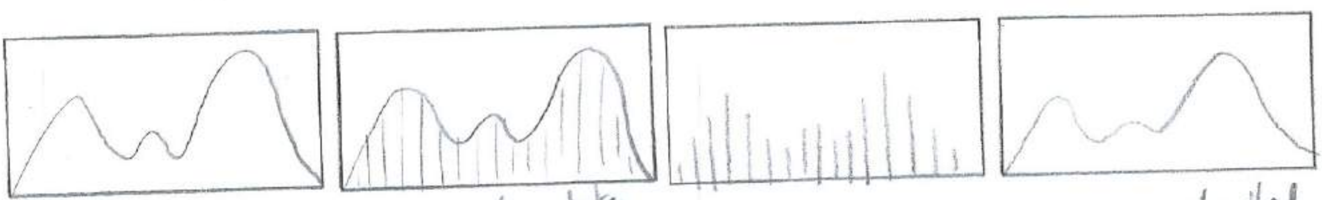
2^n = possible values it can represent

ASCII : American standard code for information interchange

8 bit code ⇒ $2^8 = 256$ possible characters

Advantages & disatvantages of digital technologies/techniques

- Conversion between Analogue & Digital forms (audio recorded to Mp3)
  - conversion of everyday analogue information
    - sample input information at regular intervals
    - Converting each sampled signal into a possible value from fixed range of values (quantum levels)
    - Convertig each sampled quantum level to digital form:
  - to increase accuracy
    - increase sample rate
    - increase available quantum levels



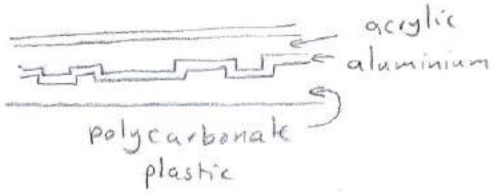
original signal | samples taken from signal | samples alon | reconstructed signal

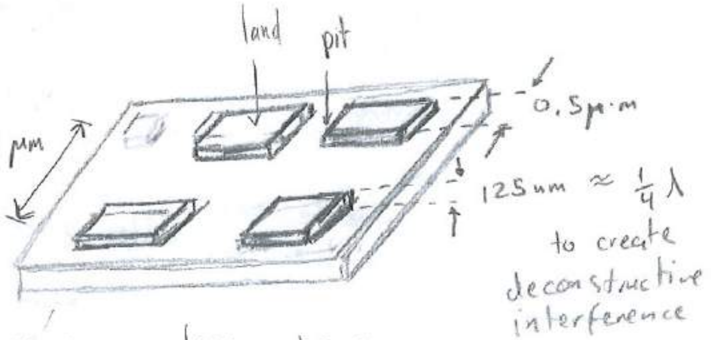
riation { - Analogue systems have large variation & noise which cannot be easily controlled or seperated from true signal.
- digital signals only has 2 values hence variation can easily be ignored.

amage { - Analogue methods of retrieval. eg LP & tape can damage recorded data
- original data is not altered

	Digital	Analogue
Complexity of code	- complex, set of rules for conversion between digital signal & out put	Simple
Quality	limits due to f & resolution. out put however can be indistinguisable from out put	Quality is virtually undistinguishable from orignal
Rreproducibility	Optical techniques allow for identical subsequent retrivals	Data prone to corruption & damage
Retrieval speed	fast	slow
Portability of stored data	small	large relative to digital
Manipulation of data	simple	complex

CD & DVD





- Info read by sensing amplitude of reflected laser beam.
- High speed of revolution when laser is reading at center compared to at the edge of CD (rate of traveling over pit = constant)
- laser beam is focused onto track
- edges cause weak signal = 1 (bit)
- Flat area causes strong signal = 0 (bit)

Capacitance

- Capacitors store charge $\dashv\vdash$ (symbol)

Charge stored = Q $\propto$ p.d across capacitor V

$\propto$ proportionality constant = capacitance C

$$C = \frac{Q}{V}$$

Q ← charge in coulombs

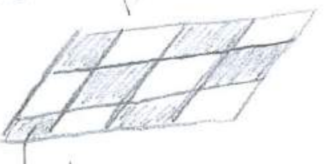
V ← p.d in volts

C ← capacitance in farads

(F) Farad is very large, capacitance measured in μF, nF, pF

$$1 F = 1CV^{-1}$$

CCD



(CCD) Charged coupled device is silicon chip used to record an image focused onto its surface.

Stages

1) during exposure each element within CCD generates a charge proportional to the incidenced light as relut of photo electric effect
   - each pixel converts light energy to electrical energy
   - more potons incidenced on pixel, more electrons omitted
2) charge is collected in different pixels
   - pixel behaves a capacitor & charge builds up.
   - charge $\propto$ # of photons (related to intensity of light)
3) charge transfered by coupling charges form on pixel to next in tern
   - signal processing takes place line by line to ensure the charge on each pixel is recorded.
4) charge packets converted individually to an output voltage & digitally recorded.
   - p.d converted to bits
   - intensity of incidenced light stored with coordinate of pixel

Quantum Efficiency

- Perfect CCD would emit one electron per photon
  - Practical CCD do no achieve this efficiency

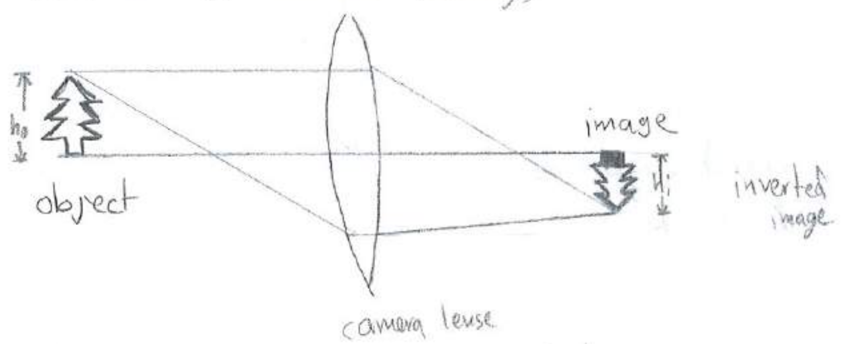
- Quantum Efficiency is the # of photo electrons emitted to the number of photons incidenced on pixel

$$QE = \frac{N_e}{N_p}$$

QE varies between 20% & 90%

Magnification & Resolution

Lenses (process of focussing)



Magnification & resolution

- Magnification is a ratio of length from lense to ccd & lense to object
- 2 images will be resolved if they are 2 pixels appart

Nuclear Structure

Isotopes

- Different nuclides that are the same element are isotopes
  - nuclides ← an atomic species characterized by specific constitution of its nucleus
  - isotopes ← same # protons - different # of neutrons

Notation

A ← mass #, # of nucleons (protons / neutrons)

X ← chemical symbol

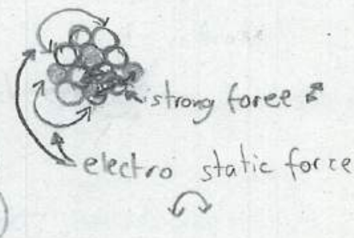
Z ← atomic #, # of protons

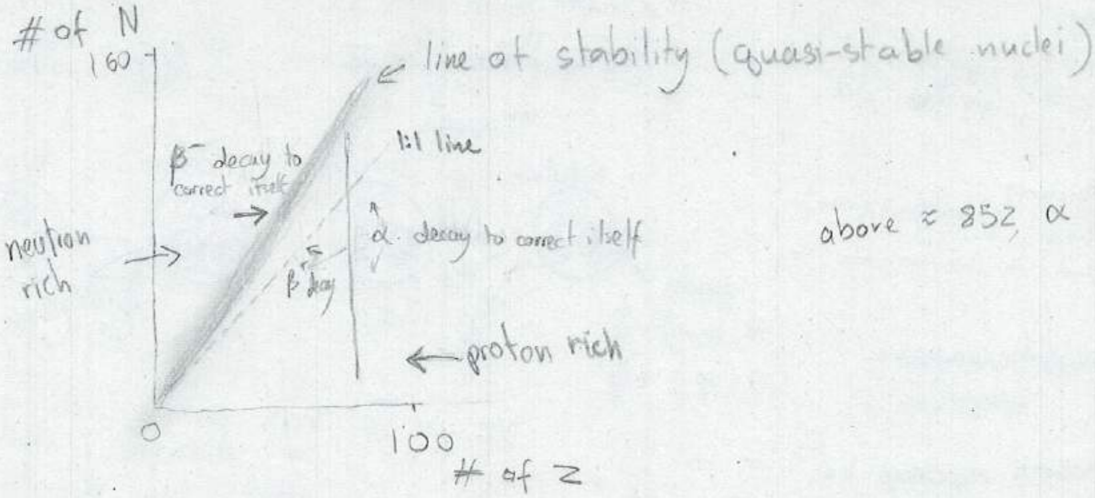
Strong Nuclear Force

- Nucleus only has positive & neutral charges - it should therefore have a force to hold the protons together (there are no negative particles to counter act the repulsion of positive charges)
  - Force called - Strong force
    - short ranged
    - very strong
    - all nucleons have it (neutrons & protons)

Nuclear Stability

- Strong force very short (only act about 10^{-35} m far, which is ≈ length of proton.) ∴ strong force only acts on adjecent nucleons, but electrostatic force can act from one end of the atom to the other. hence to balance these forces neutrons.
  - Above stability band (too many neutrons)
  - Below stability band (too many protons)
  - for larger stable nuclei # proton < # neutron





Radio activity

Types of Radiation

property / radiation	α	β	γ
effect on photographic film	yes	yes	yess
$\approx$ # of ion pairs produced per mm of air travelled (How well it ionizes)	10^4 per mm	10^2 per mm	1 per mm
typical material needed to stop/absorb radiation	10^{-2} mm aluminium (or paper)	few mm of aluminium	10cm of lead
Penetrative ability	Low	Medium	High
Path in the air $\approx$	few cm	< 1m	effectively ∞
Deflection in E & B fields (electric & magnetic), behaves like	+ charge	- charge	neutral
Speed	$10^7\ ms^{-1}$	$10^8\ ms^{-1}$ (very variable)	$3\times10^8\ ms^{-1}$

Ionizing Properties

- α, β, γ are all ionizing
- knock of electrons (interact) when passing through objects $\Rightarrow$ causes object to ionize

⚠ why radiation is dangerous. Cells DNA can be damaged $\Rightarrow$ leads to mutation.

$\Downarrow$

- ionization can mess with chemical reactions in cells called metabolic pathways
- can cause cancer

Nature of α, β, γ Decay

α - are helium nuclei

$^4_2\alpha$ (no electrons)

2 protons

e.g. $^{241}_{95}Am \rightarrow {}^{237}_{93}Np + {}^4_2\alpha$

$241 = 237 + 4$
$95 = 93 + 2$ } balances out

γ - part of electromagnetic spectrum

- after emmission nucleus has less energy but # of N or Z ore does not change.
- atom goes from excited state to a lower energy state

$$^A_ZX \rightarrow {}^A_ZX + {}^0_0\gamma$$

β - these are electrons

$^0_{-1}\beta$ (β^-)

electron e^-

$^{14}_6C \rightarrow {}^{14}_7N + {}^0_{-1}\beta + \bar{\nu}_e$

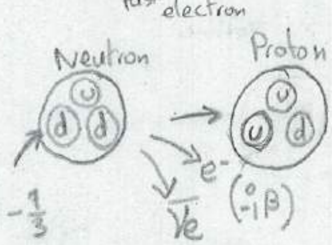
fast electron

$^0_1\beta$ (β^+)

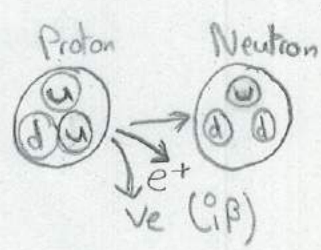
positron e^+

$^{23}_{12}Mg \rightarrow {}^{23}_{11}Na + {}^0_1\beta + \nu_e$

neutrino

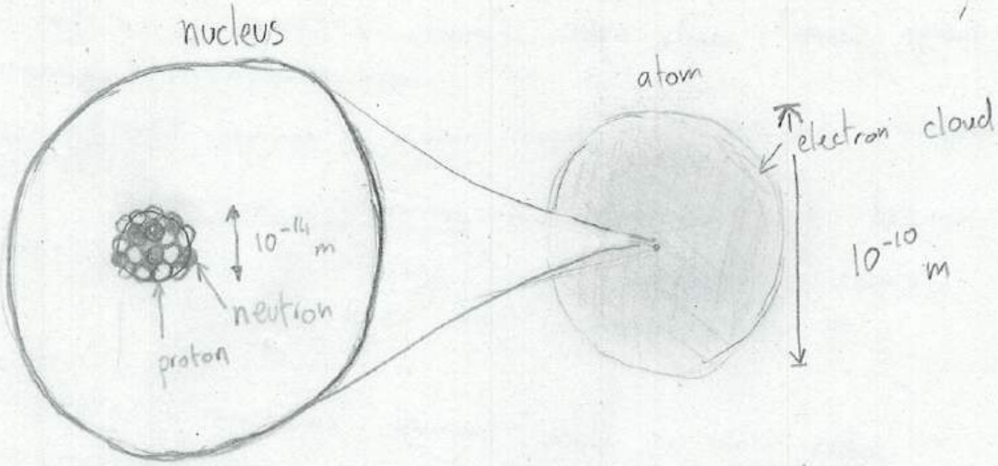


d (down) $-\frac{1}{3}$
u (up) $+\frac{2}{3}$



ν_e or $\bar{\nu}_e$ has vertually no mass so does not really affect the equation. sometimes ignored

Atomic Structure



⚠ electrons travel in circular path hence accelerate.
accelerating charges known to radiate energy,
this means electrons should loose energy
& crash into nucleus - limitation of Bohrs model)

Evidence

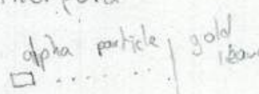
gold foil experiment.

- alpha particles shot at gold foil
- size & velocity of particles meant they were expected to travel through the gold leaf (plum pudding)
  - they were suprised to discover particles were deflected at huge angles
    1) most particles passed through the gold leaf ⇒ there must be a lot of space in the atom
    2) Alpha particles bouncing back and deflecting at angles meant there was something heavy & small.
       - could not be electrons as they were too light, also had to be positive (works as alphas were positive and bounced of this center par)

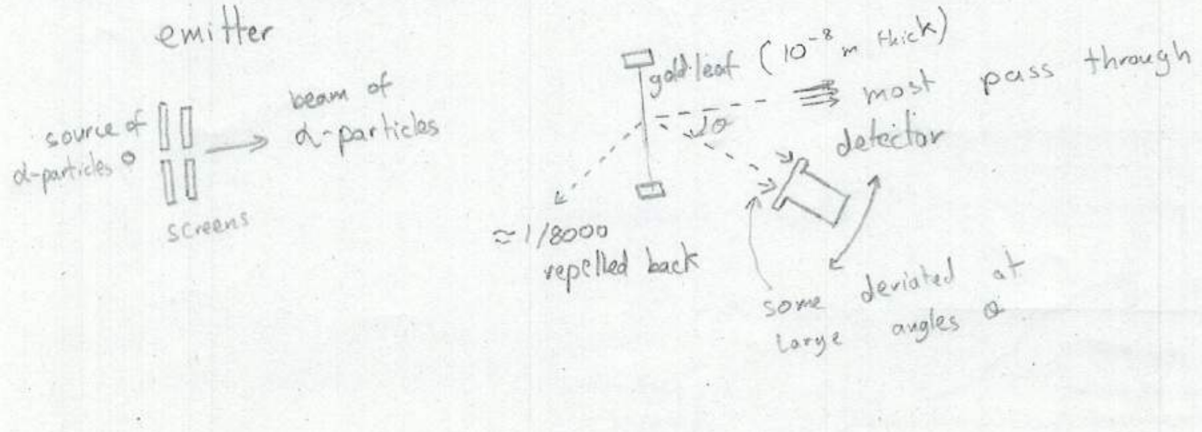
History

1) Thomson's model "plum pudding"

2) Rutherford



3) Bohrs Model

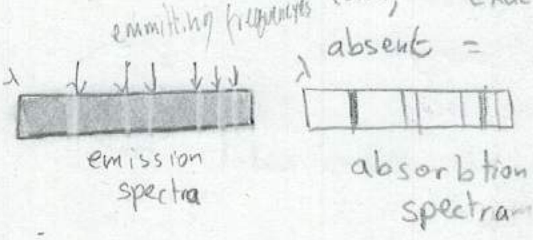


Emission / Absorption spectra

- When given enought energy element emits light (analyse the light by splitting into various colours (λ) or frequencies)
  - If all possible frequencies are present then (in analyed light) = <u>continuous spectrum</u>
  - Light emitted by elements is an <u>emission spectrum</u>
    - emission spectrum is specific to material (element)

      eg yellow - orange light sign of sodium

    - When light is shone through an element in gaseous form, exact frequencies (from emission spectrum) are absent = Absorbtion spectrum.



Explanation

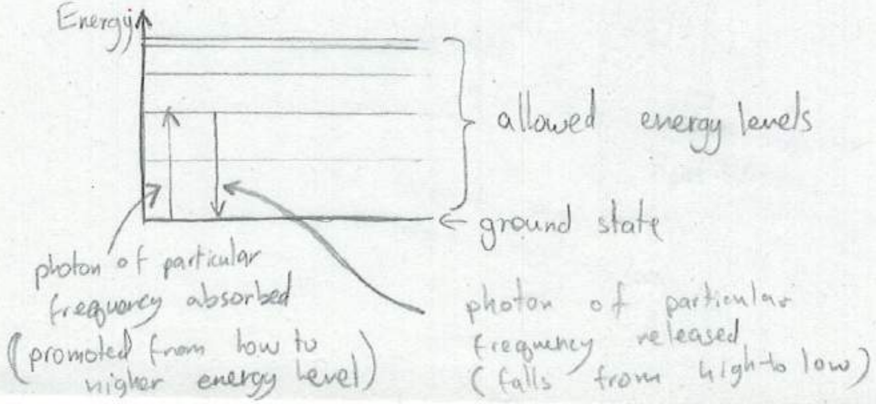
- electrons are bound to nucleus $\Rightarrow$ energy is required to make them leave the atom, which ionizes it (atom).
- Electrons can only occupy certain energy levels $\Rightarrow$ hence are said to be quantized.
  - energy levels are fixed for particular elements & depend on orbitals (more in quantum)
- Moving between energy levels electrons must emitt or absorb energy.
  - energy corresponds to Δ in energy level,
    - energy comes in packets (due to quantized levels)

$E = hf$ ← frequency in Hz

energy in joules

plancks constant 6.63×10^{-34} Js

- because energy levels are fixed, (E) energy that is released is fixed and so is frequency $\Rightarrow$ what causes emission spectra



Mass defect / Binding Energy

Binding energy: Energy released when assembling a nucleus from seperate nucleons. Initially work needs to be done to bring together neutron & proton to form nucleus <u>but</u> the created bonds between the nucleons releases <u>more</u> energy than the initial work done.

joining ⇒ release energy   seperating ⇒ input energy required

Mass Defect: Mass defect is the difference between a nucleus and the total of all individual mass.

- We know that when nucleus forms energy is released (Binding Energy)

  $E=mc^2$ (so if energy is released mass is lost)

- Bonds between the hadrons account for the difference? ⇐ <u>research this more</u>

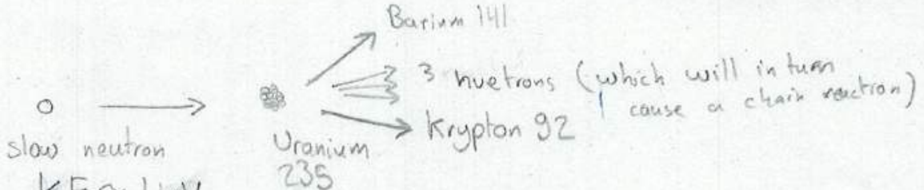
Fission / Fusion / antimater

Fission

> Large nuclei induced to break into smaller nuclei using a slow neutron

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + \text{Energy}$$

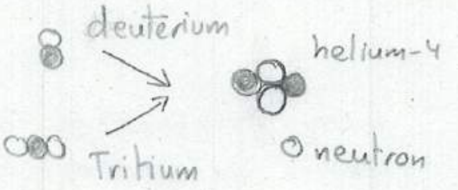
≈ 170 million eV



KE ≈ 1 eV

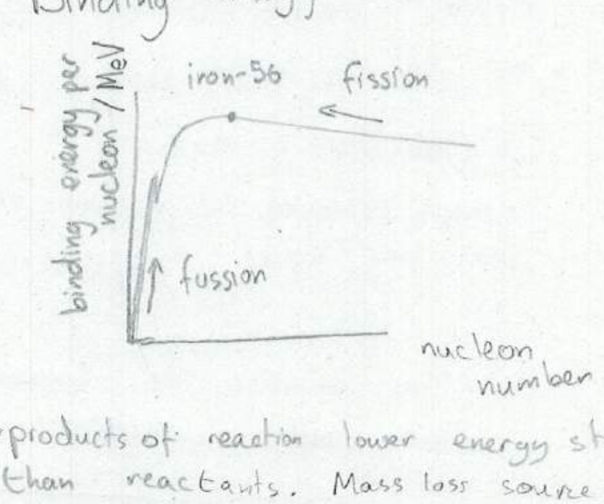
Fussion

- Nuclei are induced to join and form a larger nuclei



$${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n + \text{energy}$$

Binding Energy Per Nucleon



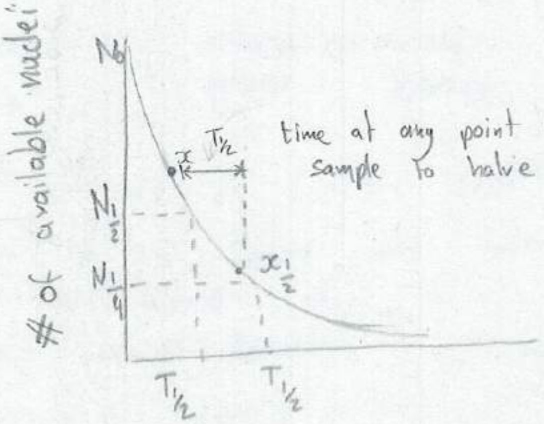
- products of reaction lower energy state than reactants. Mass loss source of energy
- to compare energy states of different nuclie binding energy per nuclei is used

Half life

- Radio active decay is random, but with large #s of atoms ~~accurate~~ predictions can be made
  - Radioactive decay is not affected by external sources
  - decay is exponential decay

Half Life

- Half life ($T_{1/2}$) can be accurately predicted due to the large number of atoms present
- Half life = time for half the same to decay (radioactively)



time at any point for sample to halve is $T_{1/2}$ (half life)

half life (time for half the sample to decay)

Nuclear Reactions

Artificial Transmutations

- Act of causing nuclear reaction (fission or fusion) by bombarding a nucleus with a nucleon, an alpha particle or any other small nucleus.

$$\underset{\alpha}{{}^{4}_{2}He^{2+}} + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}P$$

$$\left.\begin{array}{l} 4 + 14 = 17 + 1 \\ 2 + 7 = 8 + 1 \end{array}\right\} \text{balances out}$$

Units

kg

$1\ MeV_c^{-2} = 1.6 \times 10^{-13}\ J$ Energy equivalent of mass

$1\ MeV = 1.6 \times 10^{-13}\ J$ Energy

$1\ u = 931.5\ MeV = 1.66 \times 10^{-27}\ kg$ Unified atomic mass

$eV \rightarrow J\ (\times 1.6 \times 10^{-19})$

$J \rightarrow eV\ (\div 1.6 \times 10^{-19})$

Unified atomic mass:

- mass envolved in nuclear reactions ar tiny, hence physicists us U.
- U is relative to Carbon 12 which has 12 nucleons

$$U = \frac{\text{Carbon 12 mass}}{12}$$

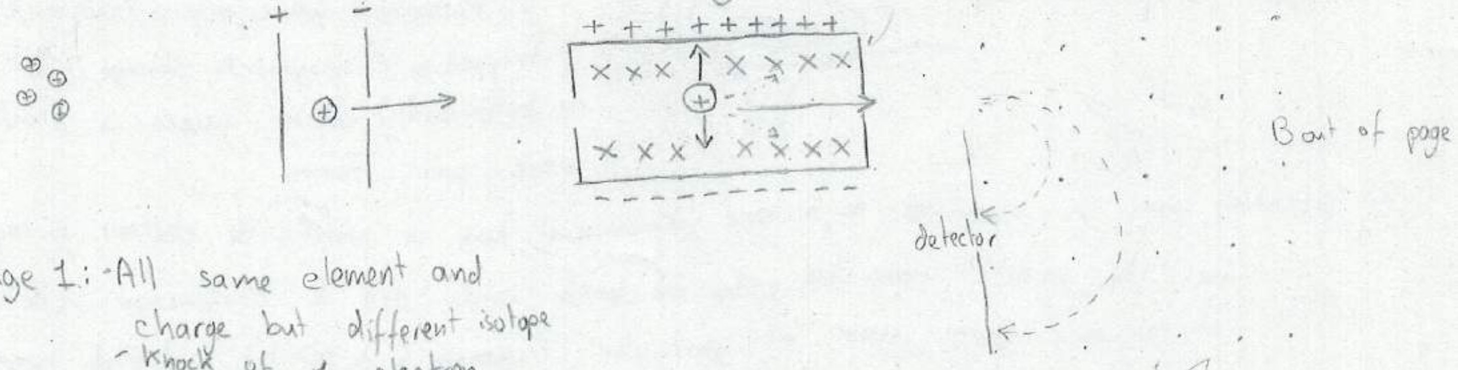
mass of proton = 1.007276u

mass of neutron = 1.008665u

mass of electron = 0.000549u

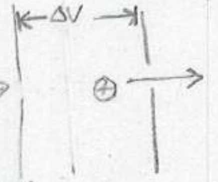
Bainbridge Mass Spectrometer

Stage 1 Ionization | Stage 2 acceleration | Stage 3 velocity Selector | Stage 4 magetic feild



Stage 1: - All same element and charge but different isotope
- Knock of 1 electron

Stage 2: Ions accelerated across plates with voltage (p.d)



we assume all ions going in have a velocity of 0.

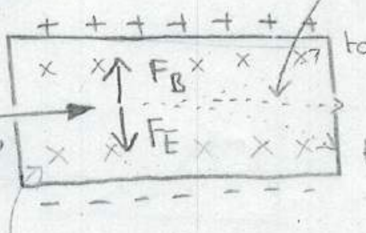
increase in KE = Loss in in EPE
↑ electro potential energy

$$\frac{1}{2}mv^2 = Q\Delta V$$

$$v = \sqrt{\frac{2Q\Delta V}{m}}$$

⇐ assuming ions enter accelerating stage with no speed. This is why a range of speeds come out of acceleration phase and velocity selector is required

Stage 3:



only if $F_B = F_E$

too fast (F_B ends up being bigger on ⊕ more v more F_B cause $F_B > F_E$)

too slow (F_E ends up being bigger on the ⊕)

⊕ with narrow range of speed

B into page

$\bullet\ F_B = QvB$ (depends on velocity hence allows us to select our speed

F_E = downward electric force experienced by positive ions in the electric feild.

F_B = upwards magnetic force

$\bullet\ F_E = QE$

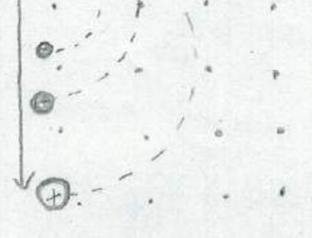
⇒ ion only will pass through if

$F_E = F_B \qquad QE = QvB$

$E = vB$

$v = \frac{E}{B}$ ← vary this to select desired speed; ← we fix this

Stage 4:



as particles are forced into uniform circular motion they gain $F_{centrapedal}$ & acceleration, hence $F=ma$ can be used to find m of particle

$qvB = F_{centripetal}$ (force acting on ions)

$qvB = \frac{mv^2}{R}$ — acceleration

$R = \frac{mv^2}{qvB} \Rightarrow R = \left(\frac{v}{qB}\right) m \Rightarrow R \propto m$

↑ this is all constant

- so we can separate & measure mass & isotopes of an element.

⚠ all magnetic fields are uniform.

The Electron in a box explained

1) electrons are not free to move outside the atom - electrons are trapped inside the potential well

- Probability wave of finding electron is trapped ⇒ (analogous to strings with nodes on both ends) which means a standing wave will form.

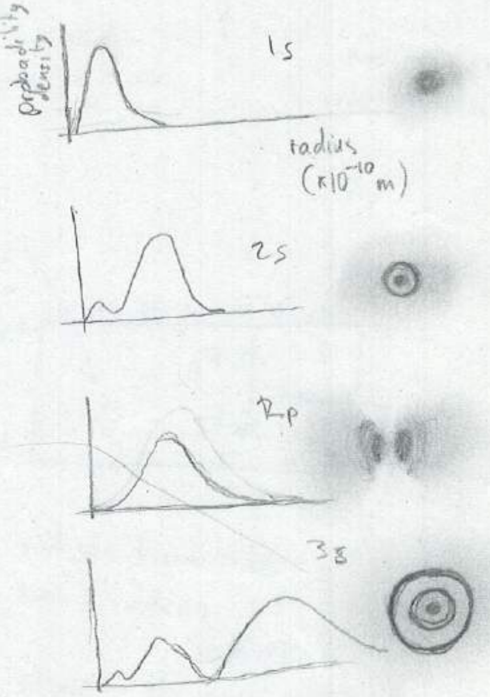
2) potential wave is simplified to a one dimensional box in which the electron is trapped.
   - as the probability wave has nodes on both ends (due to confinement in the box) it can only form wave of particular frequencies & λ ⇒ 1st, 2nd, 3rd harmonics
   - because frequencies & λ can only be certain values energy of the electrons are also fixed at certain discrete values (quantum effect)

Schrödinger model of the Atom

- Schrodinger
  - probability of finding an atom is defined by the square of the wave function ψ^2.
  - Wave fuction is not as simple as the sine curve in the 1D electron in a box.
    - The further the electron from the nucleus the lower the KE.

      KE↓ ⇒ p (momentum)↓ ⇒ use debroglie λ↑

    - ψ^2 = give prob of finding electron at a particular radius in any direction
  - Resulting orbitals ⇒ can be discribed in turms of probability of finding electrons a certain orbitals
  - When electron is observed wave function is said to collapse and complete physical particle (electron, also applies to photon) will be observed to be in one location.



- Uncertainty principle
  - Because of nature of quantum physics
    - one cannot know precisely and simultaneously
      1) momentum & position of a particle
      2) time & Energy of a particle

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

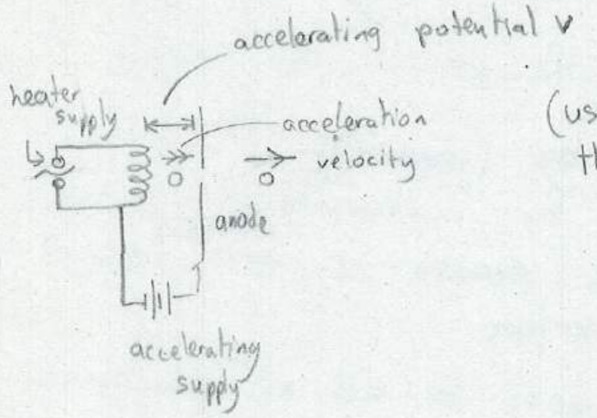
Δx: uncertainty in position; Δp: uncertainty in momentum

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

ΔE: uncertainty in Energy; Δt: uncertainty in time

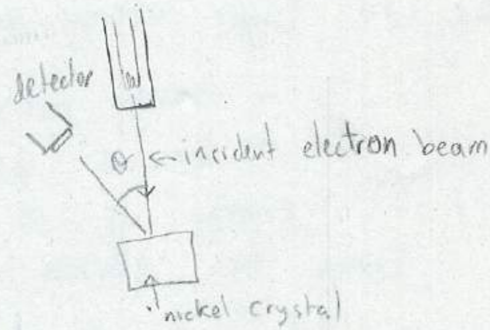
More on electrons

Electron gun



(uses principle of thermionic emmission)

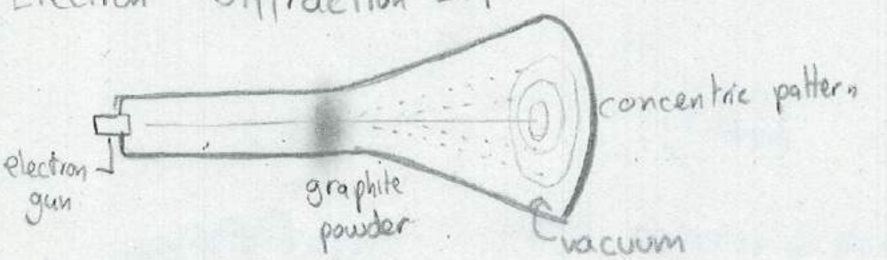
Daviso & Germer Experiment

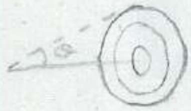


θ angle of maximum intensity of reflected electrons. The detector measured this phenomenon, it is due to constructive interference

note: - reflected in all direction
- peaks due to interference

Electron Diffraction Experiment





The circles show angle at which constructive interference takes place.

Begroglie hypothesis

↑ acceleration (from p.d in electron gun) ⇒ p to increase for each electron

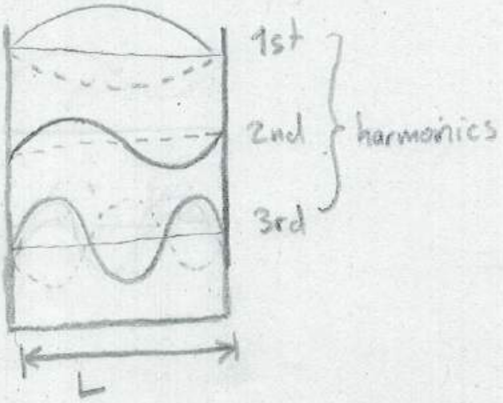
$\frac{h}{\lambda} = p \Rightarrow \lambda = \frac{h}{p}$ $\uparrow p$ $\lambda\downarrow$

as $\lambda\downarrow$ slit gap is proportionally bigger ⇒ ↓ defraction ⇒ θ of defraction↓

$\theta\downarrow$ ⇒ circles get smaller

- Waves give probability of finding electron in given space.
- Spacial nodes & spacial antinodes

Electron in a box



$\lambda = 2L, L, \frac{2L}{3},$

$\lambda = \frac{2L}{n}$ where "n" is an integer

use $\lambda = \frac{h}{p}$ to calculate momentumm in box

$p = mv$
$p^2 = m^2v^2$
$\frac{p^2}{m} = mv^2$
$\frac{p^2}{2m} = \frac{1}{2}mv^2 = KE$

$p = \frac{h}{\lambda} = \frac{nh}{2L}$

KE (E_k) of electron mass (m_e) is thus

$$E_k = \frac{1}{2}m_e v^2 = \frac{1}{2}\frac{p^2}{m_e} = \frac{1}{2m_e}\left(\frac{nh}{L2}\right)^2 = \frac{n^2h^2}{8m_eL^2}$$

Quantum

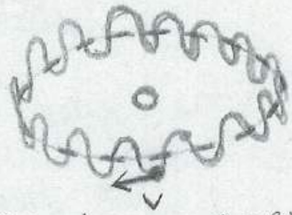
Einsteins Model : think of light as particles

- $E=hf$

1) Electrons on surface need min energy to leave energy level, work function.
2) UV Light energy arrives in lots of little packets of energy.
   - energy per packet determined by frequency
   - # of packets determined by intensity
3) If energy of a photon is large enough it gives enough Energy to electron to leave metal surface
4) Any "extra" Energy from photons goes to KE of electrons
5) If Energy from UV photons too small → still absorbed by electron but then distributed

De Broglie / Wave particle duality

- Light reflects, refracts, diffracts & interfers like waves
- Sometimes think of light as particle & sometimes as waves - wave particle duality
- Electrons have wave like properties



(Standing wave - explains why electrons don't radiate energy)

De Broglie

$E=hf$ $E=mc^2$

- A moving particles have a "matter wave" associated with them.

$hf=mc^2$ $c=f\lambda$

$hf=mcf\lambda$

$\frac{h}{\lambda}=mc$ $mv=p$ (c (speed of light))

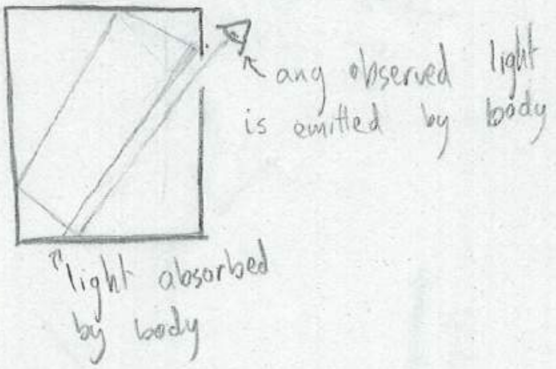
$\frac{h}{\lambda}=p \Leftarrow$ momentum

something with λ has momentum

something with momentum has a wavelength

Quantum

Black body: Object that doesn't reflect radiation, so all light observed from body is all radiated by sam body



- As you heat up black body light is emitted. The light is a function of temperature

colour of light depends on λ

$$\lambda_{peak} \propto \frac{1}{T}$$

or

$$f_{peak} \propto T$$

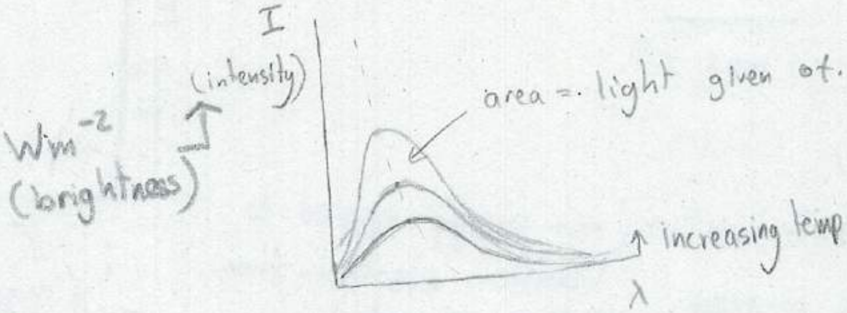
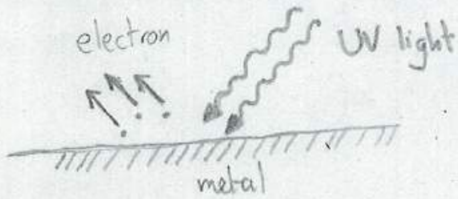


Photo electric effect

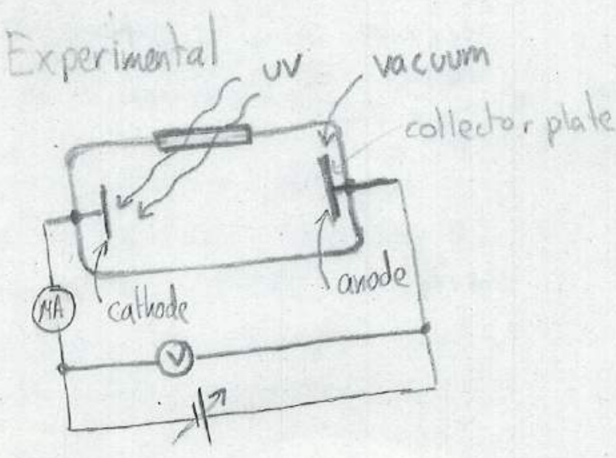


- Under certain conditions when light is shone onto metal electrons are released

- Thermionic emission (emission of electrons when metal is heated) already discovered.

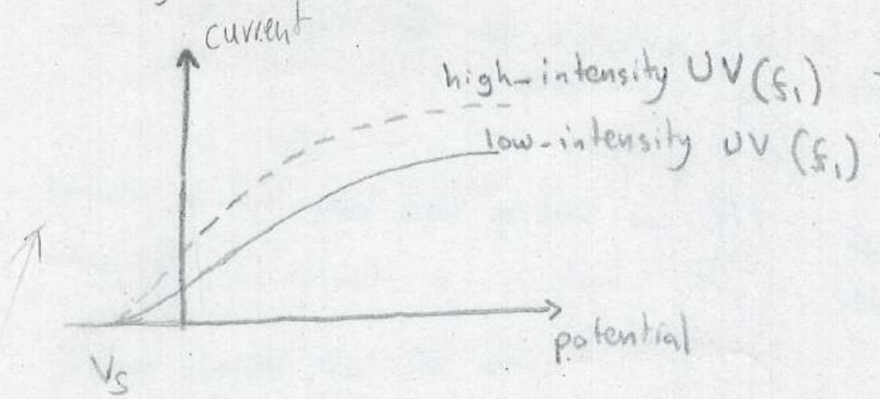
1) Below a certain frequency (<u>threshold frequency</u>), f_0, no photoelectrons are emitted.
2) Increasing frequency of light (that is above threshold frequency) increases max KE of electrons.
3) The number of electrons emitted depends on the intensity of light (brightness)
4) The effect is instantaneous.

Experimental



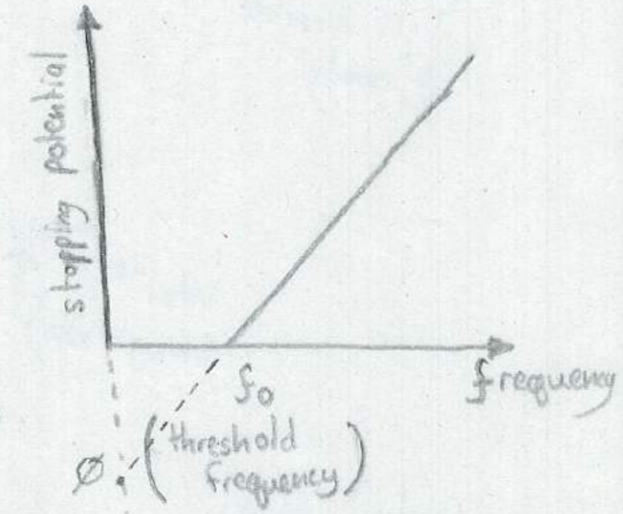
1) photo electrons released by the cathode when subject to UV light.
2) electrons accelerated by P.D to anode
3) By reversing potential between anode & cathode electron can be decelerated
4) At certain potential value, the <u>stopping potential</u>, V_s, no more photocurrent is observed. ⇒ photo electrons have been brought to rest before arriving.

Stopping potential



⚠ V_s depends on frequency of UV light

⇓

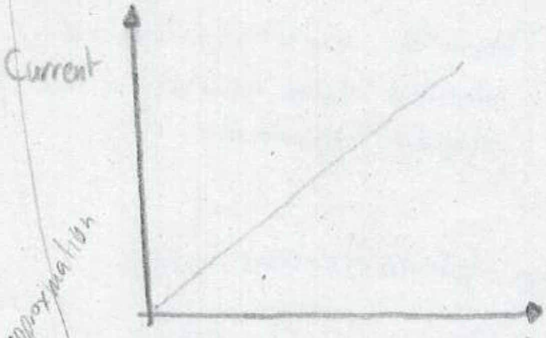


Equation for Photo-electric effect

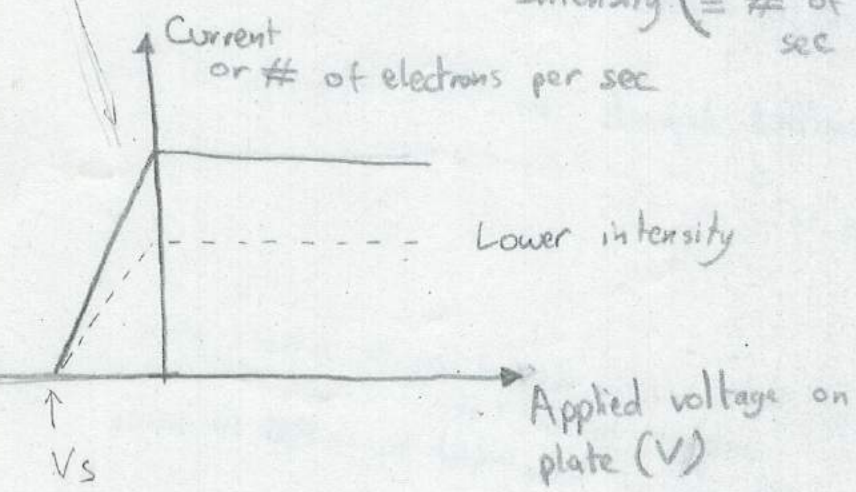
$$\underbrace{hf}_{\text{Energy carried by one quantum of light (J)}} = \overset{\text{work function}}{\varnothing} + \underbrace{\tfrac{1}{2}mv^2_{max}}_{\text{max KE}}$$

$\varnothing$ = work function (min energy required to liberate electrons from surface of metal)
↑ unique to metal

- Stopping potential = Max KE of electrons
- $\varnothing = hf_0$



⚠ only $\approx \frac{1}{10000}$ photons cause electron to be released.



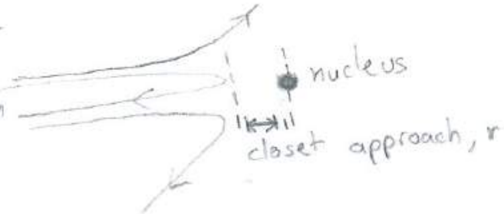
If Wave model was Correct

1) If light were a wave; energy should be supplied to all electrons. They would gather the energy continuously until they are energetic enough to leave metal
2) photo electric effect would occure at all levels
3) If amplitude of wave is increased (↑ intensity) then KE of electrons should also increase + # of electrons ↑
4) low intensity light would cause a non-spontaneous release of electrons

The Nucleus size

- Alpha particles allowed to bombard gold atoms

1) alpha particles are emitted from their source with a known energy. As they get closer to gold atom KE → EPE. Closest approach α particle stops (All EPE, no KE)



$$KE = \frac{1}{2}mv^2 \quad EPE = \frac{kQq}{r}$$

$$\frac{1}{2}mv^2 = \frac{kQq}{r}$$

charge of alpha (+2e)

radius

charge of nucleus (Ze)

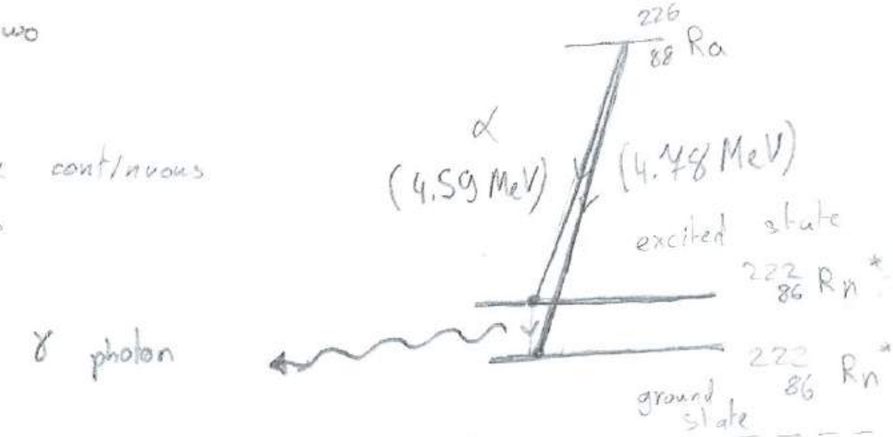
- can rearange & find r = radius of nucleus

Energy Levels

- nucleus energy levels are higher than electrons.
- Energy emitted (discrete) by α or γ decay = difference between two nuclear energy levels

note = β energy spectra are continuous because of nuetrons

decay of ^{226}Ra into ^{222}Rn



Decay Math

- Proportionality between rate of decay and # of nuclei

general equation

$$N = N_0 \left(\frac{1}{2}\right)^n$$

n ← number of half life

left radioact particle

starting particles

$$\frac{dN}{dt} = -\lambda N$$

decay constant (time^{-1} : s^{-1} yr^{-1})

probability of decay per unit time

activity (number of decay per unit time)

$$N = N_0 e^{-\lambda t}$$

final number

initial #

$$A = A_0 e^{-\lambda t}$$

$A \propto N$

Bq

$$A = \frac{dN}{dt} = -\lambda N$$

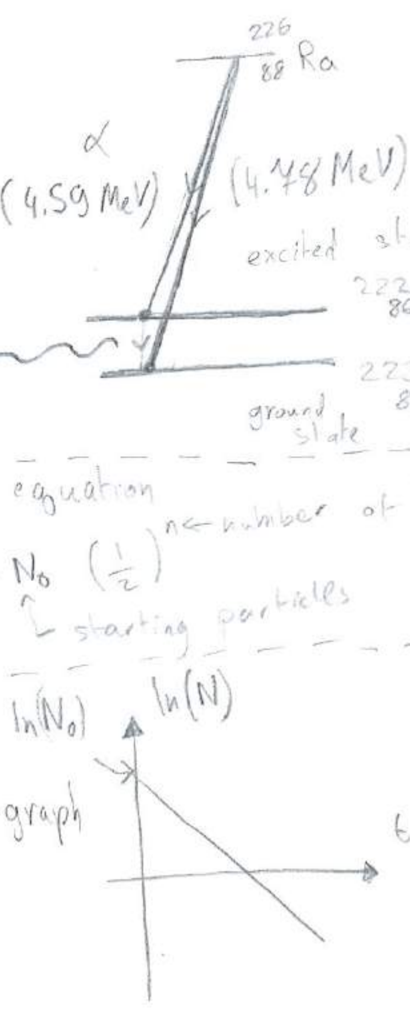
Useful to use natural logs → graph

$\ln N = \ln N_0 e^{-\lambda t}$

$\ln N = \ln N_0 + \ln e^{-\lambda t}$

$\ln N = \ln N_0 - \lambda t \ln e$

$\ln N = \ln(N_0) - \lambda t$



$$T_{1/2} = \frac{\ln 2}{\lambda}$$

Mechanics

Vector (direction & magnitude) / Scalar (magnitude)

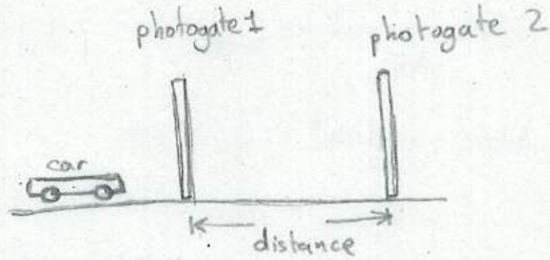
displacemant | distance

(m)

velocity = $\frac{\text{displacement}}{\text{time}}$ (ms^{-1}) → + ← −

speed = $\frac{\text{distance}}{\text{time}}$ (ms^{-1})

Instantaneous Vs Average velocity



over period of time ⇒ average velocity = $\frac{\text{distance}}{\text{time beetween photogates}}$

at a point in time ⇒ instantaneous velocity = $\frac{\text{length of car}}{\text{time through photogate}}$

↳ if increases → body is accelerating

Acceleration

- Rate of change of velocity

acceleration = $\frac{\text{change in velocity}}{\text{time}}$ = ms^{-2}

acceleration includes (changing speed. & direction).

Suvat & SI (1Dimension) (only usable in uniform acceleration)

s=displacement t = time u = initial velocity v = final velocity a = acceleration

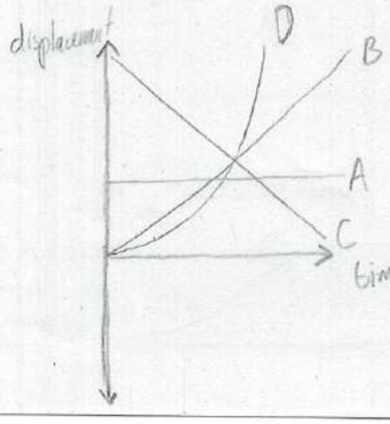
$v = u + at$ ← memorize

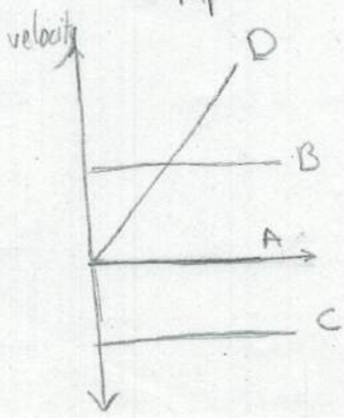
$s = ut + \frac{1}{2}at^2$

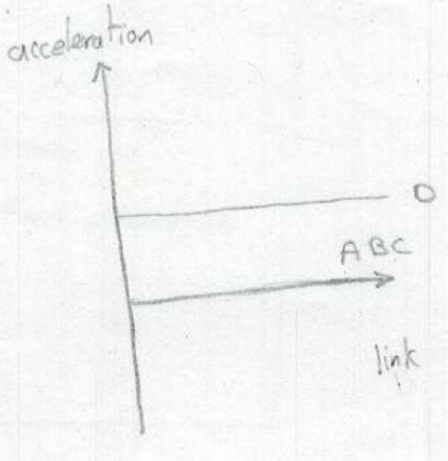
$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

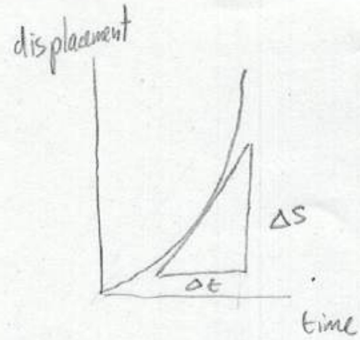
Graphs





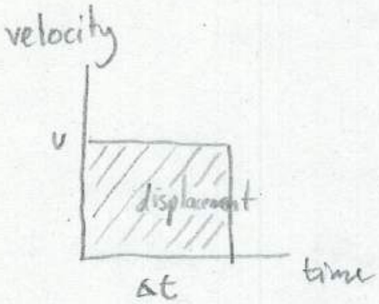


link letters together

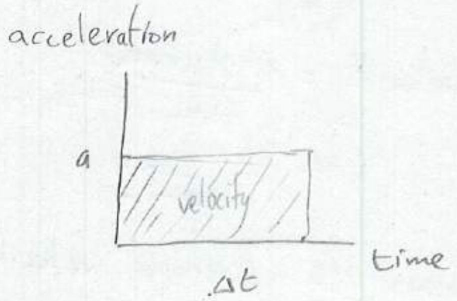


$\frac{\Delta S}{\Delta t} = \Delta v$

instantaneous v = tangent to a point on displacement time graph

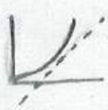


$\Delta S = v \Delta t$



$\Delta V = a \Delta t$

Note : gradient

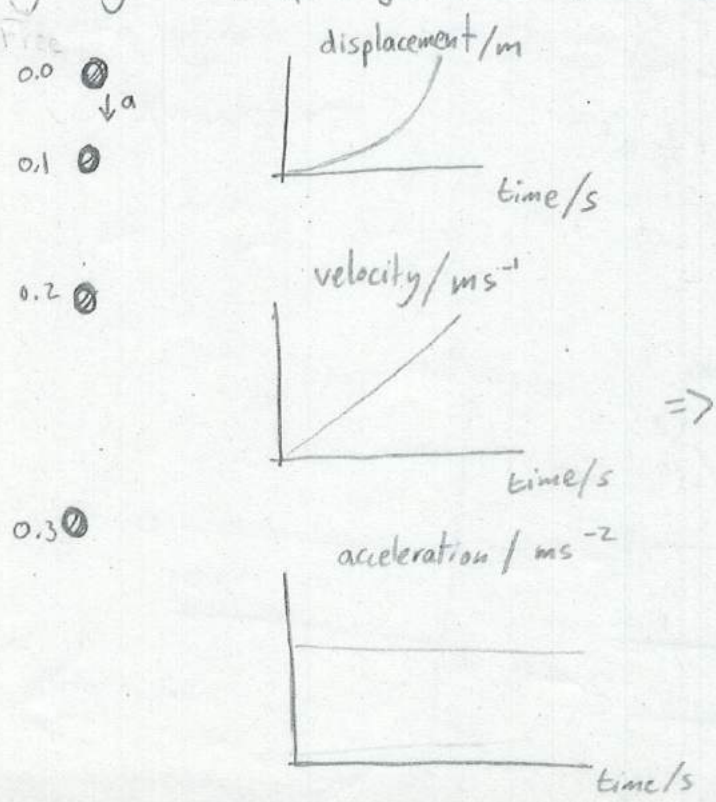
- finding straight line section of graph 
  (average value)
- finding tangent 
  (instantaneous value)

area

- Area gives integral

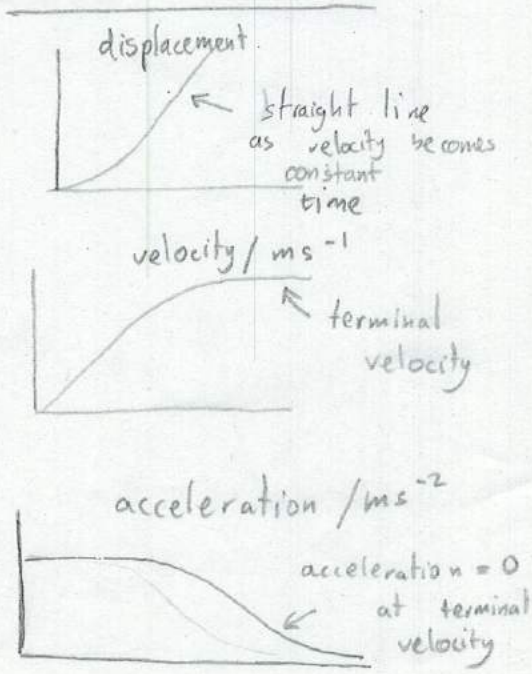
a ⇓ area = velocity / gradient = rate of change of acceleration
V ⇓ area = displacement / gradient = acceleration
S ⇓ / gradient = velocity

Falling objects (uniform gravitational field + freefall (ignoring air resistance))



=>

with air resistance



Forces & free body diagram

What is it: • the cause of $\begin{cases} \text{velocity change} \\ \text{deformation} \end{cases}$ SI unit: N

• A force (resultant) causes acceleration

deformation

velocity change

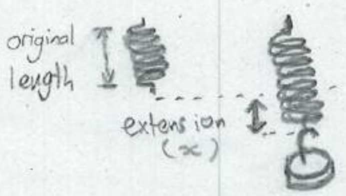
Types of Forces

- Gravitational force
- Electrostatic force
- Magnetic force
- Normal
- Friction
- Tension
- Compression
- Upthrust
- Lift

⚠ origin of all these forces is either gravitational or electromagnetic

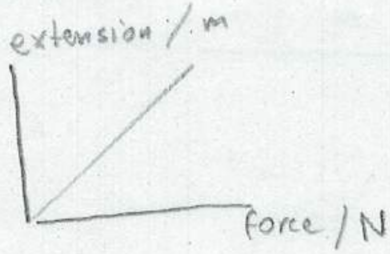
Hooke's Law

States: Up to an elastic limit, the extension, x, of a spring is proportional to tension force, F. The constant of proportionality k is called the spring constat



$F \propto x$

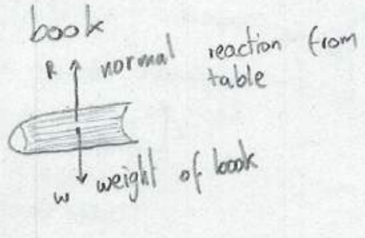
$F = kx$

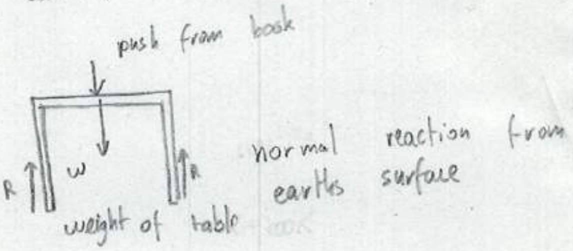


Free body diagram

1) only one object is chosen
2) all forces on object are shown & labelled

book on table





Newtons' Laws

① An object continues uniform motion in a straight line unless acted opon by an external resultant force.

⇒ object won't accelerate unless there is a resultant force.

Equilibrium:

- object with zero resultant force is said to be in translational equilibrium
  mathematically $\Sigma F = 0$

Using Newtons 1st law we know object is in equilibrium if:

1) object at rest
2) object moving uniformly in a straigh lin
   (velocity)

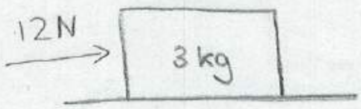
⚠ being at rest ≠ system under equilibrium (think of pendulum)
instantaneously at rest by system is not in translational equilibrium.

② The resultant force is proportional to the rate of change of momentum

$$F = \frac{\Delta p}{\Delta t} \Rightarrow F = ma$$

mass kg (m), acceleration ms^{-2} (a), resultant force N (F)

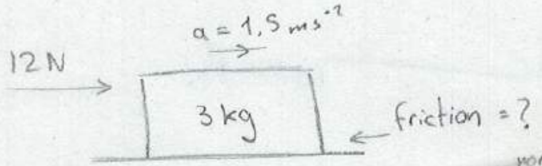
Examples



$F = 12$
$m = 3$
$a = ?$

$F = ma$
$\frac{F}{m} = a$
$\frac{12}{3} = 4$

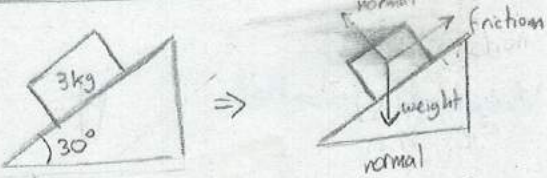
$\therefore a = 4 ms^{-2}$



$m = 3$
$a = 1.5$
$F = ?$

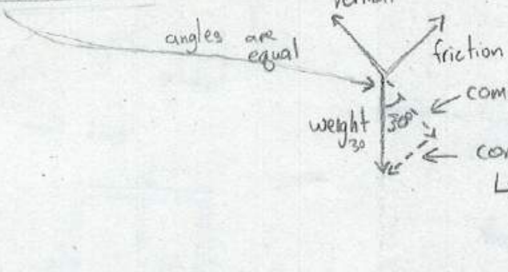
$3 \times 1.5 = 4.5$
$F = 4.5$

resultant force = forward − friction
∴ friction = forward − resultant
$12 - 4.5 = 7.5N$



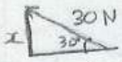
weight ≈ 30N
friction = 8N (given)
normal = ?

acceleration = ?



← component into slope (= normal)

← component down slope (used for acceleration)

↳ 

$30\sin(30) = x = 15N$
$15N - \text{friction} = 7N$
$F = ma \quad a = \frac{f}{m} \quad a = \frac{7}{3} = 2.3 ms^{-2}$

③ Every action has an equal but opposite reaction

$F_{AB} = -F_{BA}$

- The 2 forces act on different objects
  ⚠ equal & opposite reactions that act on 1 object are not newton's 3rd Law
- Forces acting must be of same type.

Mass vs Weight

mass: amount of matter contained in an object (kg)
weight: force exerted on a body by gravity (N)

mass $\propto$ weight

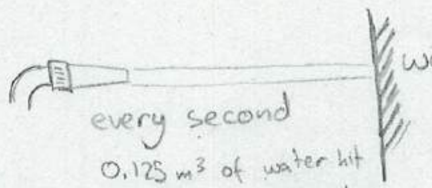
Momentum

Symbol: p $p = mv$ SI unit $kg\,ms^{-1}$ or Ns since velocity is vector so is momentum

Δp is the impulse $\Delta p = F\Delta t$ force appied over time its appied

in second Law $\dfrac{\Delta p}{\Delta t} = F$

Impulse $= m(v-u)$



every second
$0.125\,m^3$ of water hit wall at $50ms^{-1}$

$0.125\,m^3 \Rightarrow 125\,kg$

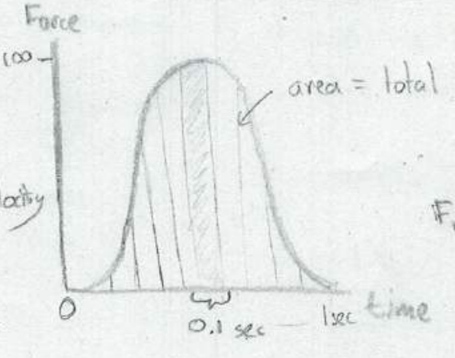
momentum $= 125 \times 50 = 6250\ kg\,ms^{-1}$

water brought to rest $\therefore\ \Delta p = 6250\ kg\,ms^{-1}$

$\text{Force} = \dfrac{\Delta p}{\Delta t} = \dfrac{6250}{1} = 6250\,N$

---

ball being kicked
$500g \Rightarrow 0.5\,kg$
what is the final velocity of the ball.



area of graph = impulse $\approx F_{max}\,\Delta t \cdot \frac{1}{2}$

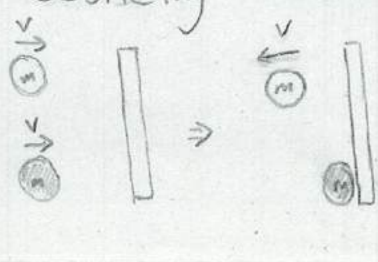
$F_{max} = 100 \quad \Delta t = 1$

$\Delta p = \frac{1}{2} \times 100 \times 1 = 50\,N$

$\Delta p = m\Delta v$

$\dfrac{\Delta p}{m} = \Delta v = \dfrac{50}{0.5} = 25\ ms^{-1}$

ball bouncing



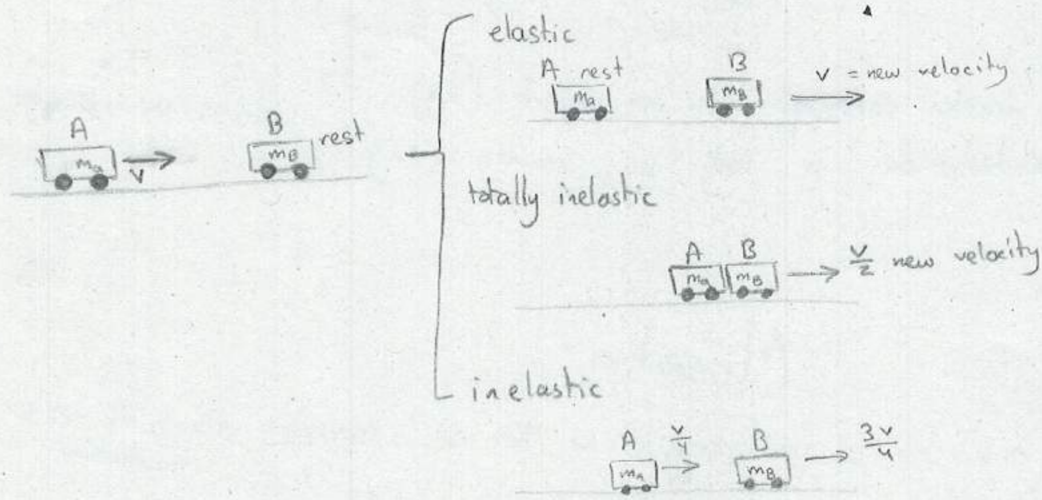
clear ball
p before $= mv$
p after $= -mv$ (vector)
$\Delta p = p\text{ after} - p\text{ befor} = -mv - mv = 2mv$

dark ball
p before $= mv$
p after $= 0$
$\Delta p = -mv$

Conservation of momentum

- The total momentum of a system remains constant provided there is no resultant external force.

Collisions



all KE passed onto cart B, no energy loss occures, momentum is conserved

large amount of mechanical energy lost but total momentum conserved

between totally inelastic & elastic, momentum is conserved

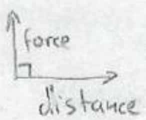
Work

Work done = force x distance travelled in direction of force

SI Nm : newton meters or J : Joules scalar

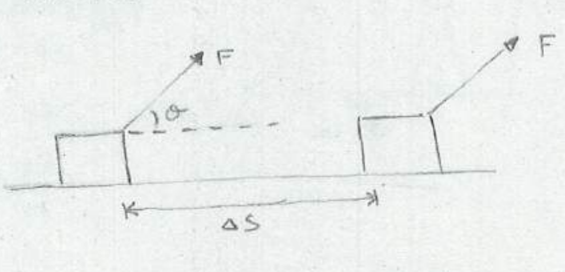
⚠ - you need a force (resultant) acting, constant velocity in a frictionless system does not count.

- If force is perpendicular to direction of movement no work is done



$F \cos 90 \cdot \text{distance} = 0$

general formula



$F \cos \theta \, \Delta S =$ work done

Energy & Power

Work & energy

- when object does work it gains energy (object doing work on another object loses energy)

Conservation/Transformation of energy

- principle of conservation of energy → overall total energy of any <u>closed</u> system must be constant
  → Energy is neither created nor destroyed, it is only transfered
  → There is no change in energy in the Universe

Types

Kinetic
Gravitational potential
Elastic potential
Electrostatic potential
Thermal
Electrical
Chemical
Nuclear
Internal
Radiant
Solar
Light

Kinetic $= \frac{1}{2}mv^2$ potential (gravitational) $= mgh$ Elastic potential $= \frac{1}{2}kx^2$

k = spring constat
x = extension

Power

Power = rate of transfer of energy

SI $= Js^{-1} = W$

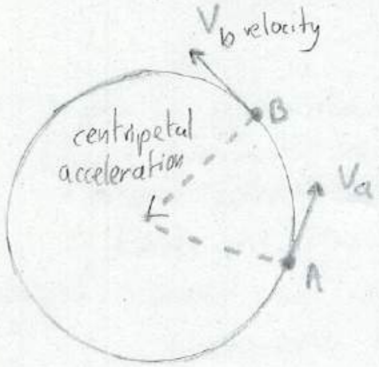
$$\text{Power} = \frac{\text{energy transferred}}{\text{time taken}} = \frac{\text{work done}}{\text{time taken}}$$

$$\text{Efficiency} = \frac{\text{useful work/energy/power OUT}}{\text{useful work/energy/power IN}}$$

Circular Motion

- speed constant but because of changing direction velocity is changing ⇒ acceleration is changing
- acceleration in circular motion = **centripetal acceleration**

centripetal force ⇒ causes acceleration



$|V_a| = |V_b|$ magnitude is the same

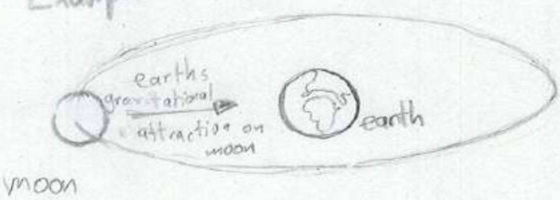
direction of change in velocity is towards center of the circle (acceleration)

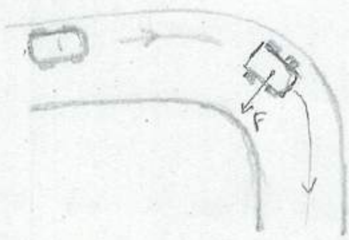
$$a_{centripetal} = \frac{v^2}{r}$$

$$f_{centripetal} = m\, a_{centripetal}$$

⚠ centripetal force does <u>not</u> do any work (force & direction of mouvement at 90° ⇒ no work)

Examples





Projectile Motion

<u>projectile motion</u> traveling in a <u>parabola</u>

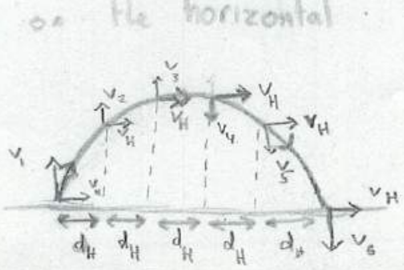
- only forces acting { gravity (vertical component), friction (horizontal component) ← not considered usually }
- horizontal & vertical components are independant so they can be split

Horizontal

- velocity constant (only in horizontal) here no forces are acting on the horizontal component

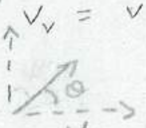
Vertical

- There is a constant vertical force (resultant), this is usually gravity.



Projectile motion continued

- Use the angle of launch to resolve any initial velocity into components $V_V = v \sin\theta$



$V_H = v\cos\theta$

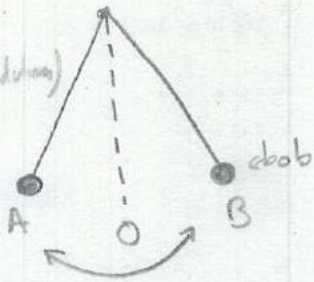
- time of flight will be determined by vertical component of velocity
- range of flight is determined by horizontal component & time of flight

note: · greatest range acheived at 45° launch angle
· 2 objects released  both hit ground symultaneously

Waves & SHM

Cycle: One cycle is one complete oscillation of wave (pendulum) (A-B-A). Also referred to as a complete circle in circular motion.

Pendulum



⚠ SHM: special type of oscillation that occurs under these mechanical conditions
- system is at rest in some position
- there is a restoring force towards the equilibrium
- SHM gives sinusoidal relationship.

Equilibrium position
- position bob will rest if not disturbed. (O)

Amplitude (x_0)
- maximum displacement from equilibrium position (OB or OA)

Time Period (T)
- time to complete 1 cycle

Frequency (f)
- number of cycles per unit time.

Units: s^{-1} or Hz $\quad f = \frac{1}{T}$

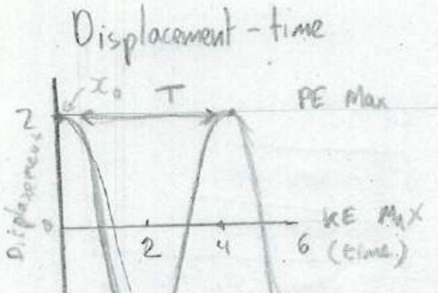
Angular frequency (ω)

$\omega = 2\pi f$ or $\omega = \frac{2\pi}{T}$ $\quad$ Units: s^{-1} or Hz

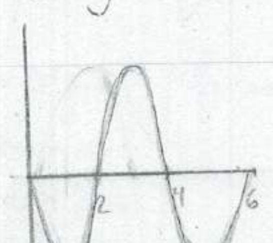
2π rads s^{-1} = one rotation per second

Restoring force
- Force acts towards equilibrium & proportional to the displacement from the equilibrium.

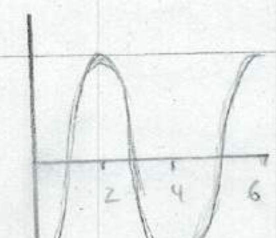
Displacement - time

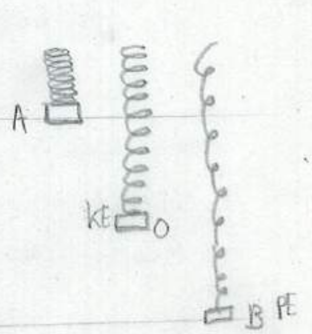


Velocity-time



acceleration - time





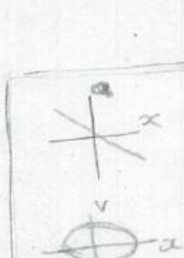
$x = x_0 \cos \omega t$ (x: displacement; ω: $2\pi f$)

$v = -v_0 \sin \omega t$ (v: velocity; v_0: max velocity; ω: $2\pi f$)

$a = -a_0 \cos \omega t$ (a: acceleration; a_0: max acceleration; ω: $2\pi f$)

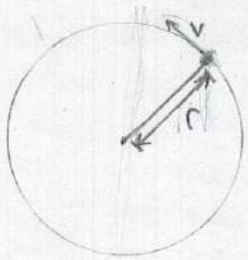
$a \propto -x$

non calculus



Equations for SHM

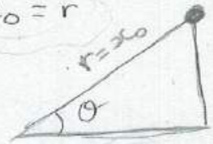
calculus



⚠ horizontal component of circular motion is SHM

Displacement

$x_0 = r$



$x_0 \sin\theta$ ⇒

$x_0 \cos\theta$ ← we want horizontal component

$x = x_0 \cos\theta$

$v = \dfrac{\text{distance}}{\text{time}}$

distance $= 2\pi r$

$v = \dfrac{2\pi r}{T}$

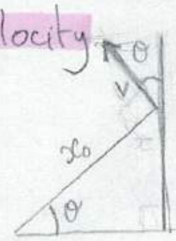
$\dfrac{2\pi}{T} = \omega$

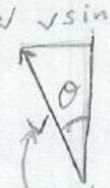
$v = \omega r$

$r = x_0$

$v = \omega x_0$

Velocity





$v\sin\theta$ horizontal velocity

$v\cos\theta$

circular motion

horizontal velocity $= -v\sin\theta$

⟵ $v = \omega r$ (in circular motion)

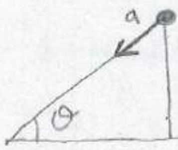
$v = -\omega x_0 \sin\theta$

Acceleration

acceleration (towards center) when in circular motion

centrapetal acceleration $= \dfrac{v^2}{r} = \dfrac{(\omega r)^2}{r} = \dfrac{\omega^2 r^2}{r} = \omega^2 r$

since $r = x_0$ $a = \omega^2 x_0$



$\sin a$

$\cos a$ ← we want horizontal component

horizontal acceleration $= -a \cdot \cos\theta$

$= -\omega^2 x_0 \cos\theta$

Displacement $= x_0 \cos\omega t$ ↲ differenciate

Velocity $= -\omega x_0 \sin\omega t$ ↲

Acceleration $= -\omega^2 x_0 \cos\omega t$ ↲

we also learned ↙ Mazorica

$a = -\omega^2 x$

$v = \omega\sqrt{x_0^2 - x^2}$ ⟸ if we have given displacement

$V_0 = \omega x_0$

$\sin^2\theta + \cos^2\theta = 1$

$\sin\omega t = \sqrt{1 - \cos^2\omega t}$

$\omega x_0 \sin\omega t = \omega x_0 \sqrt{1 - \cos^2\omega t}$

$\omega x_0 \sin\omega t = \omega\sqrt{x_0^2 - \underbrace{x_0^2\cos^2\omega t}_{x^2}}$

$\omega x_0 \sin\omega t = \omega\sqrt{x_0^2 - x^2}$

$V = \omega\sqrt{x_0^2 - x^2}$

when $x_0 = 0$ $v = V_0$

$V_0 = \omega x_0$

Energy SHM

KE

$v = \omega\sqrt{x_0^2 - x^2}$

KE by definition $= \frac{1}{2}mv^2$

$\therefore$ $KE = \frac{1}{2}m\omega^2(x_0^2 - x^2)$

$KE_{max} = \frac{1}{2}m\omega^2(x_0^2 - 0^2)$

$KE_{max} = \frac{1}{2}m\omega^2 x_0^2$

$KE_{max} = PE\ (zero)$

$PE_{max} = KE\ (zero)$

KE max at equilibrium point during cycle where $x = 0$

Total Energy

total energy at any moment $= PE + KE$

so when KE_{max} (bottom of swing) total energy $= \frac{1}{2}m\omega^2 x_0^2 + 0$

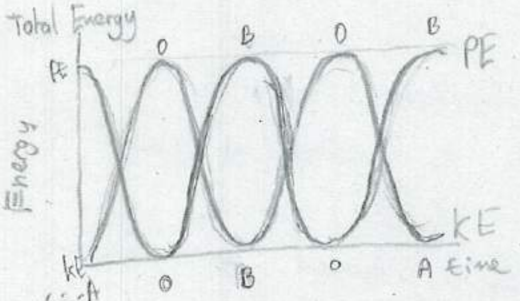
$\therefore$ total energy in a system can be found usind

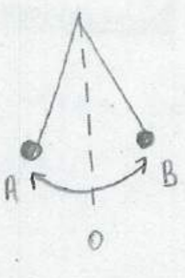
Total $E = \frac{1}{2}m\omega^2 x_0^2$

Potential energy

Total energy $-$ KE $=$

$PE = \frac{1}{2}m\omega^2 \underline{x_0^2} - \frac{1}{2}m\omega^2(x_0^2 - x^2) = \frac{1}{2}m\omega^2 \underline{x^2}$





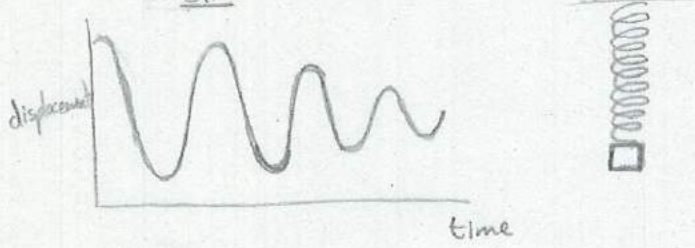
remember $1 = \cos^2\theta + \sin^2\theta$

Damping

- Damping = Air resistance, Drag, friction
(system has to work against forces resulting in energy loss)

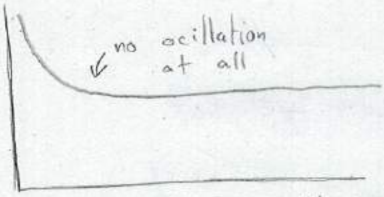
Light damping

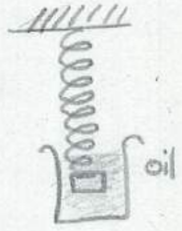
air



Critical

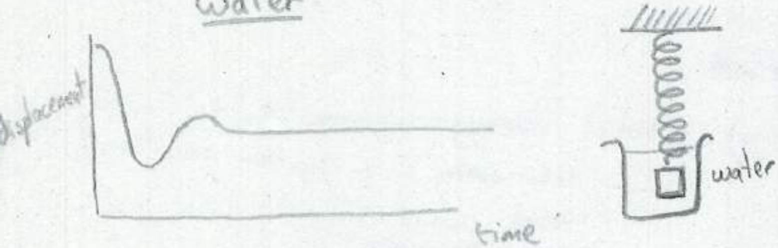
oil





returns to equilibrium without passing through it

water



air < water < oil
damping (rapidity of loss of energy)

⚠ Damping does not affect frequency but amplitude & velocity

Resonance

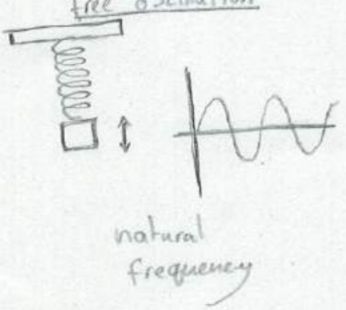
Natural frequency - frequency under which a system oscillates naturally

Forced oscillation - Other frequency system is made to oscillate at that is not natural frequency.

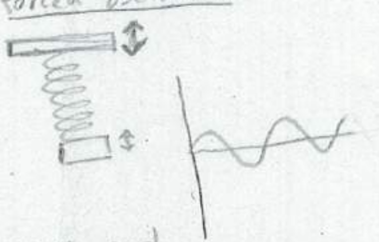
Resonance - Increase in amplitude when system is forced to oscillate at natural frequency.

note: If driving frequency = natural frequency ⇒ resonance occurs (driving frequency = frequency of driving force)
forced motion in tune with ↑

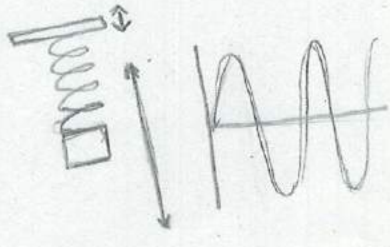
free oscillation



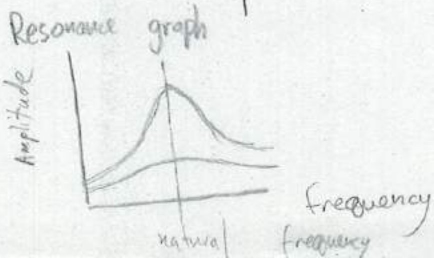
forced oscillation



resonance



Resonance graph

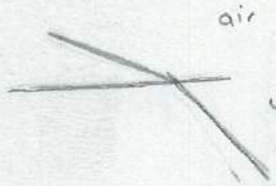


Wave Characteristics

Reflection

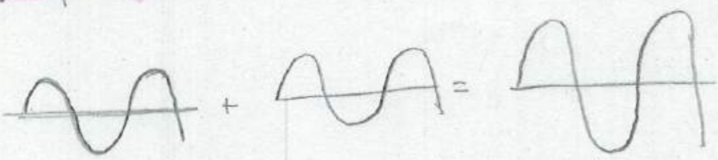
when a wave hits a wall it reflects

Refraction



when waves will bend when medium is changed

Interference



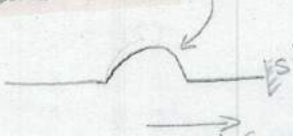
waves can add up when crossed over

Diffraction

wave spread out

Waves in String

Wave Pulse

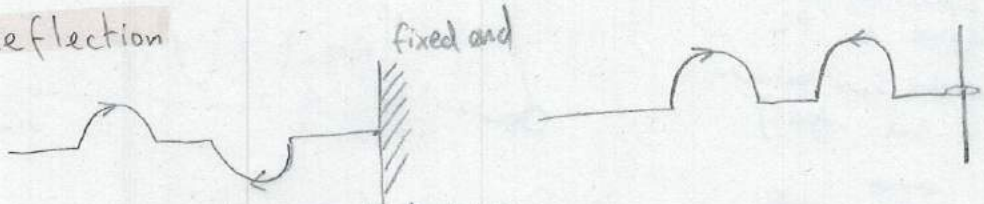


string (flicked so wave is created)

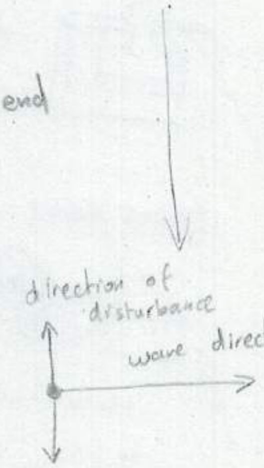
speed (wave speed)

⚠ no part of string moves in the direction of wave speed.
particles move 90° to wave speed. ↕

Reflection

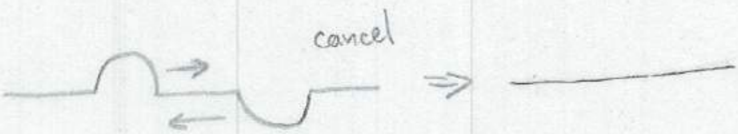


1) wave exerpts upwards force on wall
2) (newtons 3rd law) wall exerts downwards force on wave and pushes it back.
3) creates inverted reflected pulse back.

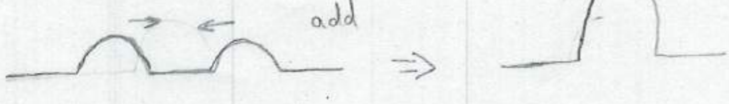


(transverse wave)

Interference



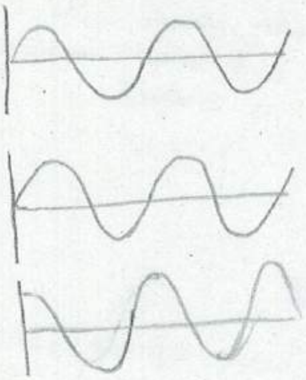
⚠ amplitude that is changed!
deconstructive interference



constructive interference

- Wave are a transfere of energy
- as string is lifted it gains PE ⇒ then PE is tranfered around the string.

Phase



in phase

out of phase ($\frac{\pi}{2}$ out of phase)

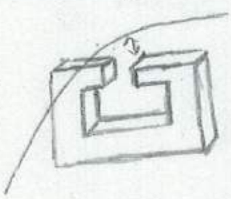
$\frac{\pi}{2}$ slightly out of phase

π out of phase

2π in phase

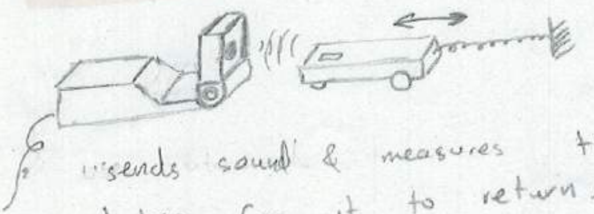
Measuring

Photo gate



detects every time object passes through & send signal to PC
(Time period = time between 1st & 3rd signal)

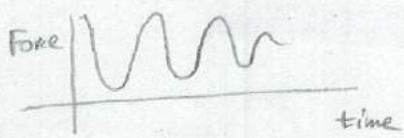
Position Sensor



sends sound & measures time it takes for it to return $\Rightarrow$ can calculate the distance between object and itself

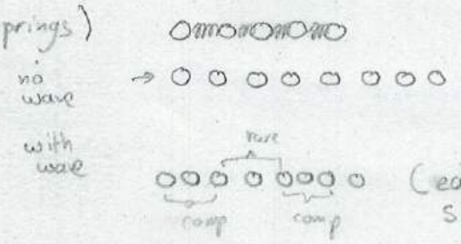
Force Sensor

Tension in string varies with time & this is measured



Representation:
Longitudinal waves

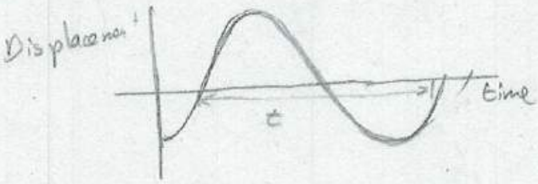
- (consider chain of balls connected by springs)



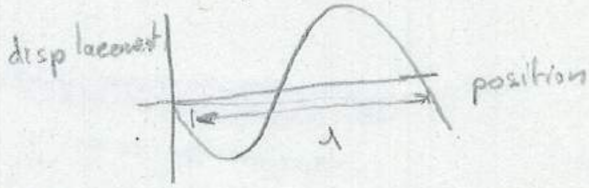
(each ball moves with shm)

Displacement - time/position

Displacement-time - for one ball

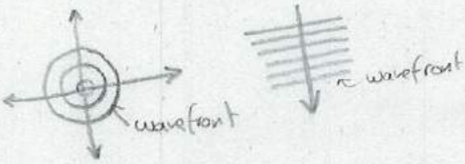


Displacement position - position of all balls in a moment in time relative to original position

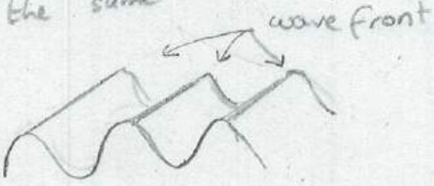


Wave Properties

Wave front:

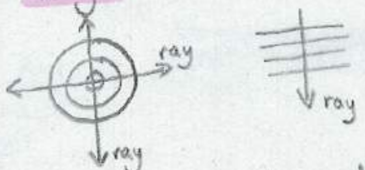


Line joining points with same displacement at the same moment. (in phase)



Circular wave fronts:
- produced by point disturbance
- forms rays that are radial

Rays



Lines that show direction of wave fronts. Perpendicular to wavefront

Plane wavefronts
- produced by extended disturbance
- far away point: circle appear straight

Strings (changes & how they affet v, λ, f)

(High pitch = High frequency)

1) Thick string = low pitch :

speed inversly proportional to mass

mass ↑ $v\downarrow$ — λ must stay constant $\therefore f\downarrow$

$v\downarrow = \lambda f\downarrow$

$f\downarrow = \frac{v\downarrow}{\lambda}$

2) short string = high pitch

length of string = λ

$\downarrow$ string length = $\downarrow\lambda$

$\uparrow f = \frac{v}{\lambda\downarrow}$

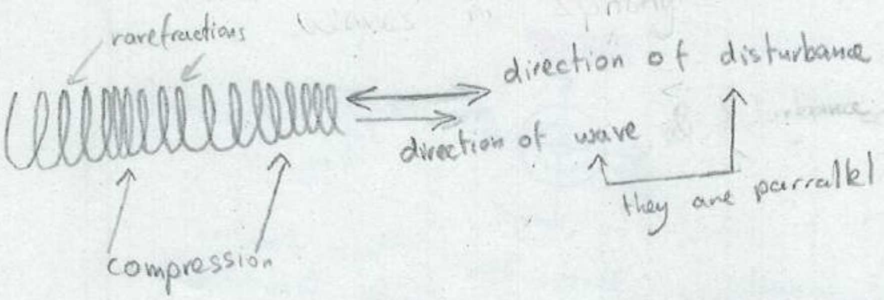
3) tight string = high pitch

tension $\propto$ velocity

tension ↑ velocity ↑

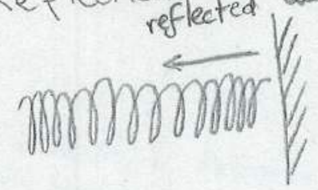
$\uparrow f = \frac{v\uparrow}{\lambda}$

Springs



(longitudinal wave)

Reflection



Interference

displacemen super imposes as it does in transverse waves

Distinguish between longitudinal & Transverse waves

1) only transverse waves can be polarised

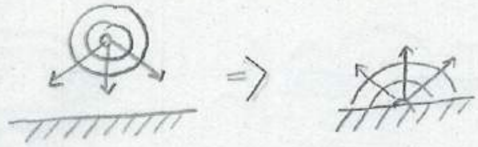
⚠ **Displacement-time/ position** (representation of transverse waves)

Displacement - time
- 1 particle & it SHM movement (position at any given time)
  - time period

Displacement - position
- All particles in the position they are in the wave at one point in time
  - wave length

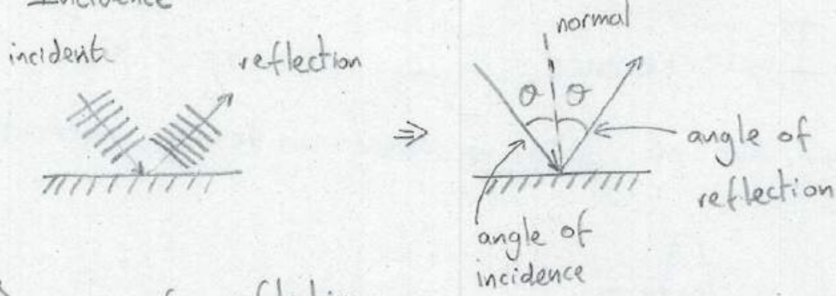
Reflection

Wave hit barrier ⇒ gets reflected



⚠ reflection appear to originate from behind the barrier = effect of mirror.

Incidence



⚠ reflection only changes the direction of the wave.

Laws of reflection

① angle of incidence = angle of refraction

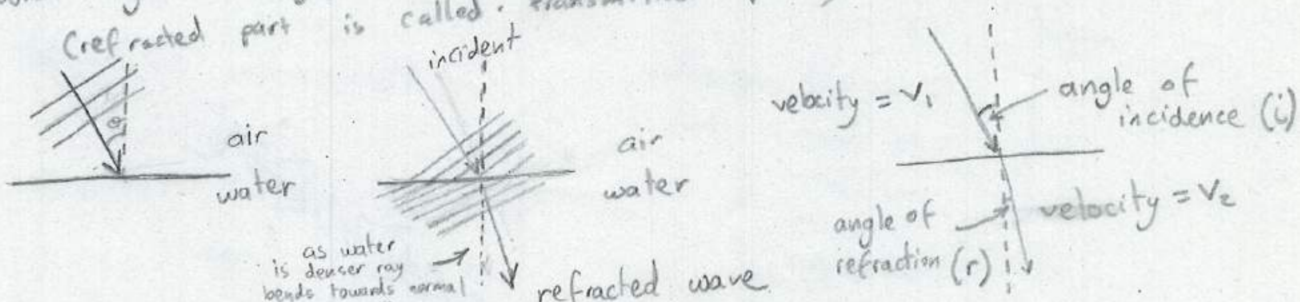
② incident & reflected ray are on the same plane as the normal

Refraction

Refraction occurs when wave changes medium

- wave changes velocity
- wave changes wave length
- wave does not change frequency or Energy would change ($E = hf$)
- If wave comes at an angle then wave changes direction

⚠ when light changes medium some is reflected some is refracted (refracted part is called: transmitted part)



Snell's Law

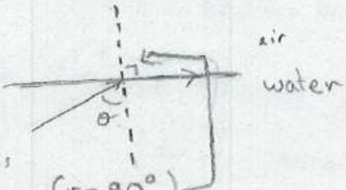
$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

refractive index of medium 2

refractive index of medium 1

refractive index: $\dfrac{\text{speed of light in vacum}}{\text{speed of light in medium}} = \dfrac{c}{v}$

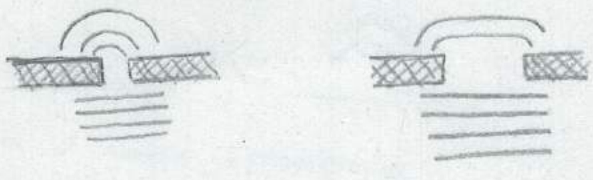
critical angle:



critical angle occures when $\sin r = 1$ ($r = 90°$)

⚠ critical angle forms when travelling through dense to light medium

Defraction

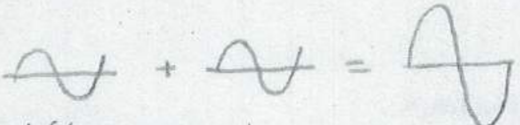


level of defraction depends on the size of slit relative to λ.

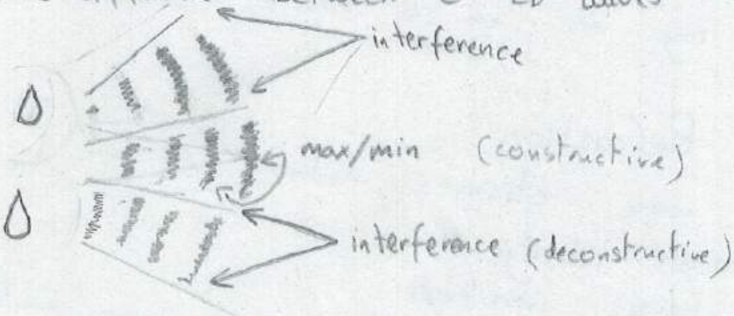
⚠ only rays change

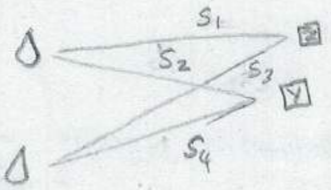
Interference

1D, wave are in phase or out of phase → hence uniformly constructive or de constructive



2D, phase difference between 2 2D waves is different at different points





$S_1 \neq S_3$ (out of phase if path difference = π)

$S_2 = S_4$ (const ructive interference)

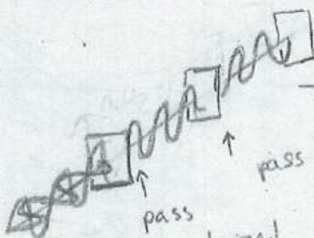
Phase angle: 1) completly out of phase = π

2) can be calculated using path difference (d) [phase angle $\varphi = \frac{2\pi d}{\lambda}$]

Light

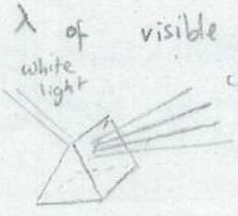
- Double slit defraction
- as waves overlap th is:

Polarization of light



λ & amplitude of light

- Light is EM - propagation of transverse disturbance in an electrical & magnetic field
  - can travel through vaccum
- λ of visible light 400 nm to 800 nm (λ determines color)



(different λ causes different angles of refraction, hence all colours become visible)

Spectrum

micro wave

visible light

wavel gamma | x-rays | UV-rays | | IR-rays | radar | FM | TV | Short wave | AM

λ 10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 1 10^{2}

Hz 10^{22} 10^{20} 10^{18} 10^{16} 10^{14} 10^{12} 10^{10} 10^{8} 10^{6}

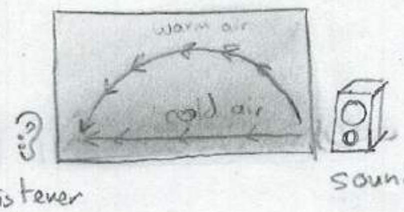
Amplitude of light related to brightness (measured using <u>intensity</u>)

$$\text{intensity} \propto \text{Amplitude}^2$$

Velocity of EM wave = $3.0 \times 10^{8}\ ms^{-1}$ (in vacuum)

Sound

- reflected in form of eco
- Refraction



warm air = fast moving particles carry sound faster

$1°C \uparrow$ increases speed of sound by $\approx 0.6\ ms^{-1}$

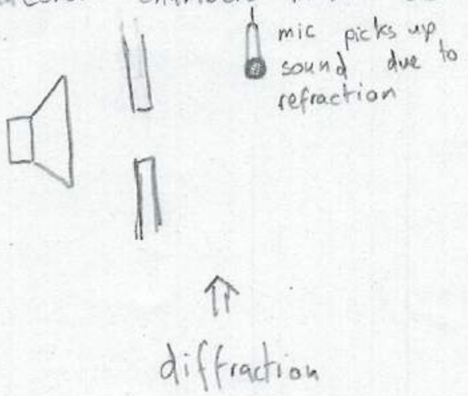
water carries sound 4x faster

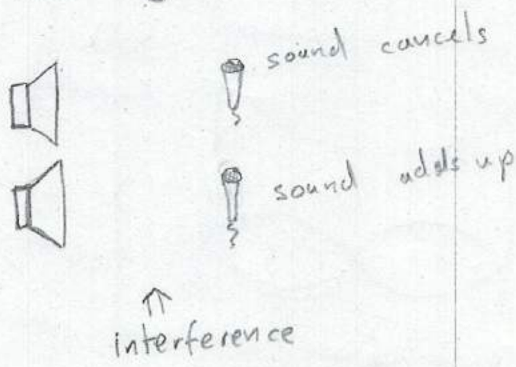
steel caries sound 15x faster

warm air causes refraction,
hense sound travels through 2 paths = extra loud

Diffraction & interference

- anechoic chambers must be used to observe anything





⚠ sound is a longitudinal wave (use wave theory to predict behaviour)

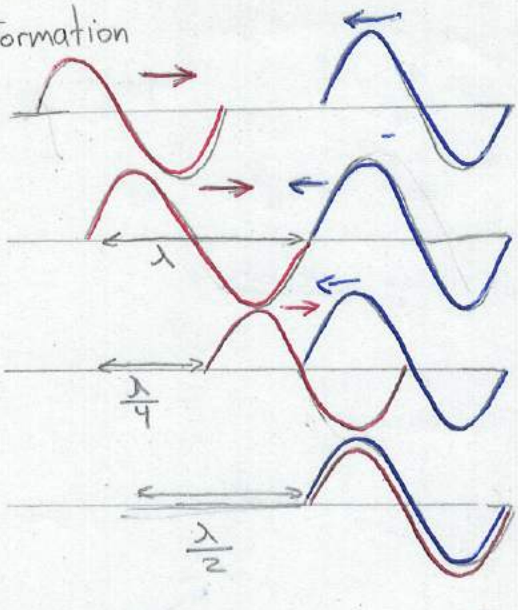
propagation of disturbance in air pressur

Frequency & amplitude

- Frequency gives pitch
- loudness is given from amplitude

Standing waves

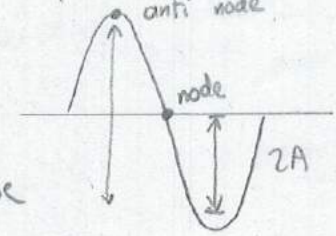
Formation



2 identical waves travel along strings on opposite ends

(2 fixed ends, blue wave is reflected wave)

out of phase



in phase

- node does not move
- anti node = max & min points (peaks & troughs)

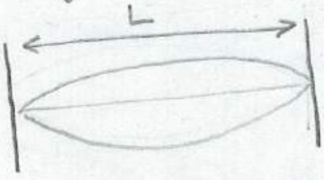
amplitude doubles

Progressive wave Vs Standing wave

Progressive wave (moving wave)	Standing wave
• progresses along the rope	• profile does not progress along the rope
• All points shorter than 1 λ are out of phase	• points between nodes are in phase (the points more than 1 node away are out of phase by π)
• All points have same amplitude - net energy flow	• some point A=0 some A=A - no net flow of energy

Stringed instrument

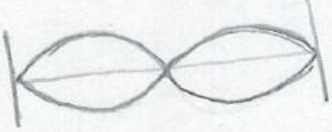
1st harmonic



$\lambda = 2L$

frequency = f_1

2nd harmonic



$\lambda = L$

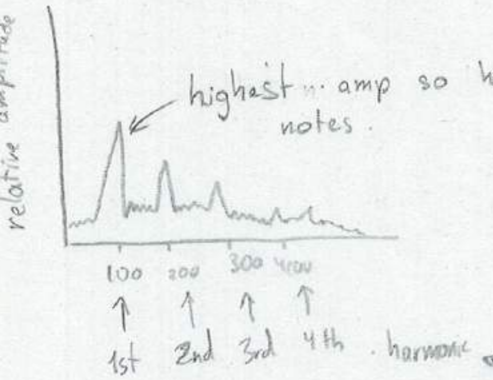
frequency = $2f_1$

1st harmonic (wave at lowest possible frequency)

use $f_1 = \frac{v}{2L}$

2nd $f_2 = \frac{v}{L}$ as v is constant we can deduce $2f_1 = f_2$

Guitar



here you will hear the first harmonic.

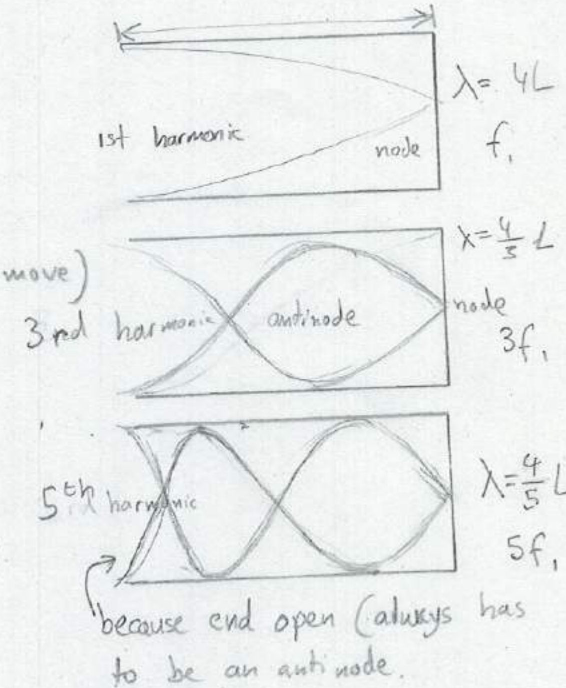
- when string is plucked all possible standing waves are produced.
- the standing waves are only made from certain frequencies (notice standing wave, surounded by nodes)

factors that affect note

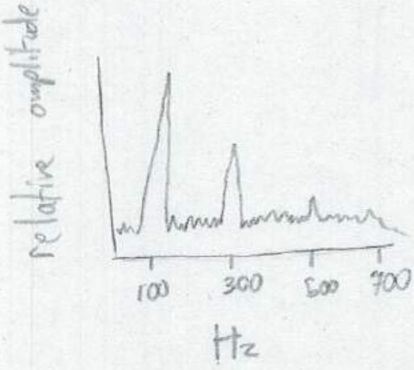
- mass of string (thickness)
- tightness
- position of nodes (made by finger)

Standing Waves in closed pipes

1) Sound wave hits closed end & is reflected
2) original & reflected wave superimpos to give standing wave
3) as sound wave is disturbance in air pressure end of pipe becomes node (air cannot move)



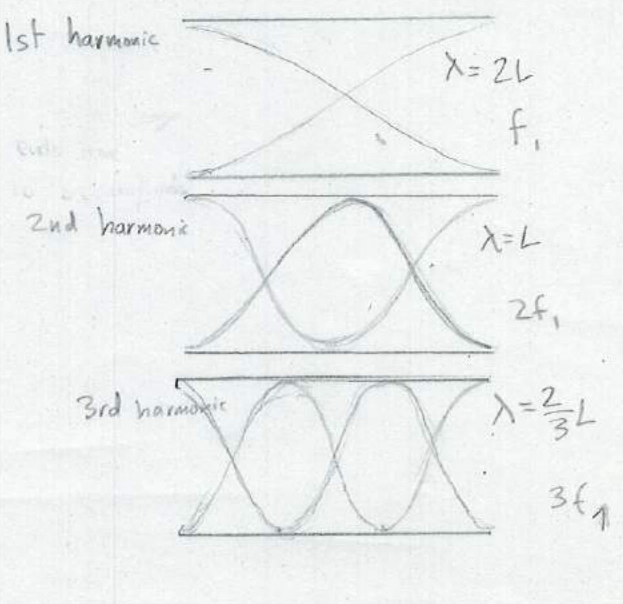
because end open (always has to be an antinode.



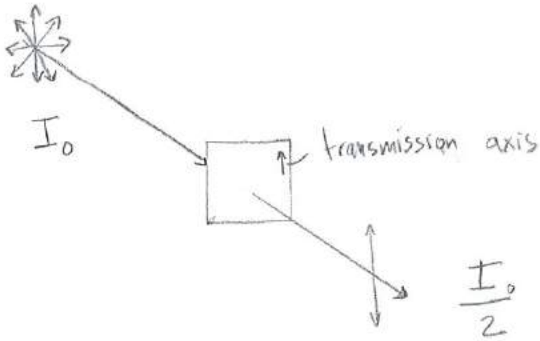
only odd harmonics are formed.

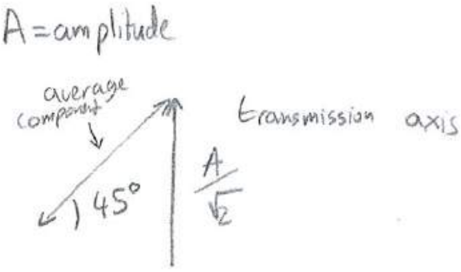
Standing wave in open pipe

- both ends are antinodes



Doppler Effect



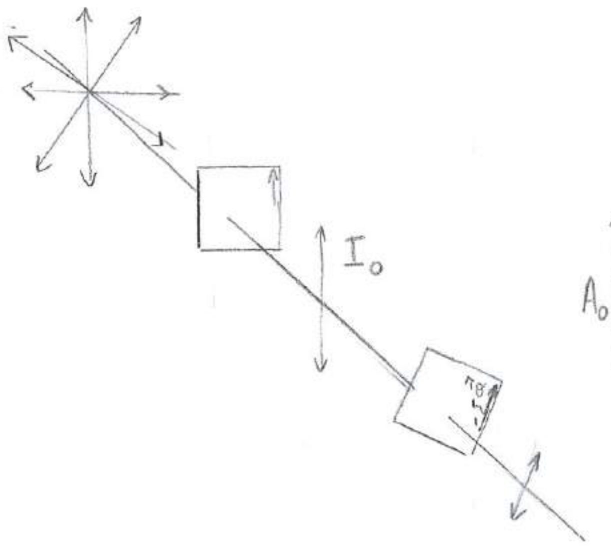


$A^2 \approx I$

$\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} = I$

Malus Law

second polarising lense called analyser

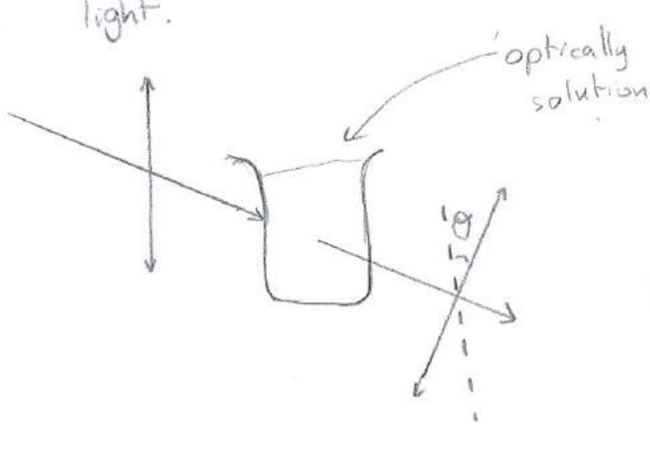


$I_t \propto A_0^2 \cos^2\theta$

$I = I_0 \cos^2\theta$

Optically active solutions

Optically active solutions rotate the plane of plarization of light.



θ depends on :

- concentration
- distance travelled in liquid

non invasive way of measuring concentration.

Improving resolution

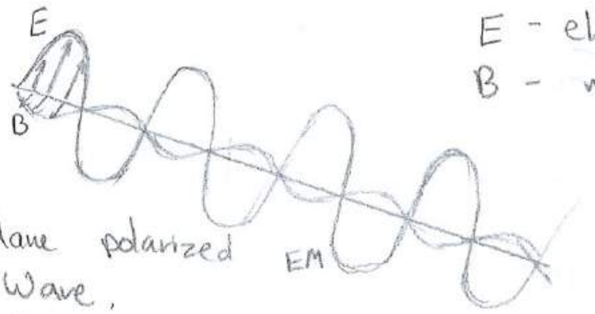
- change size of apertune
- change wave length

CDs
- make pits smaller, & shorter λ laser

Microscope (electron)
- with visible light resolve to 200nm
  - if we used shorter λ than we would go out of visible spectrum
  - That why we use election microscope which uses high energy elections with λ of 0.02 nm. We can resolve up to 0.1 nm.
    - image is detected by censors & formed using computer

Radio Telescope
- $\lambda \approx 20cm$ so telescope must be very big.

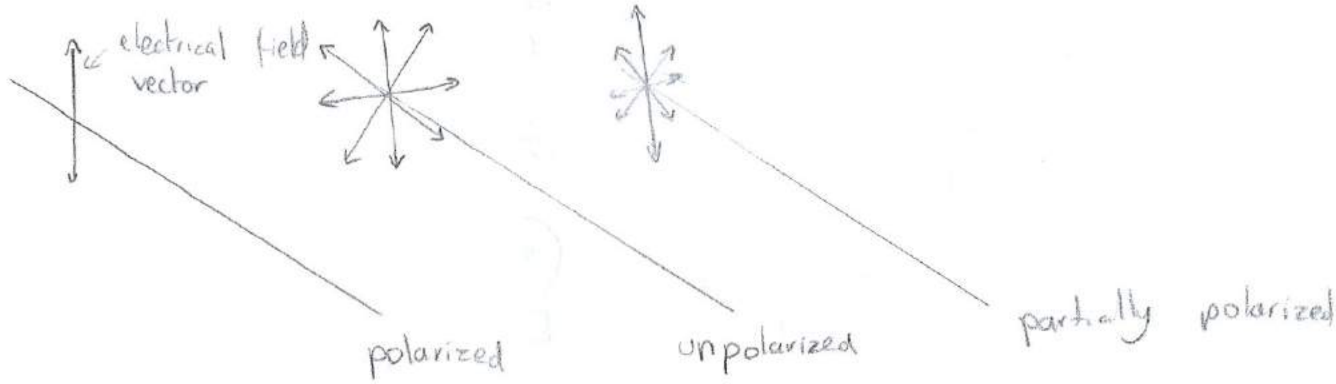
Polarization



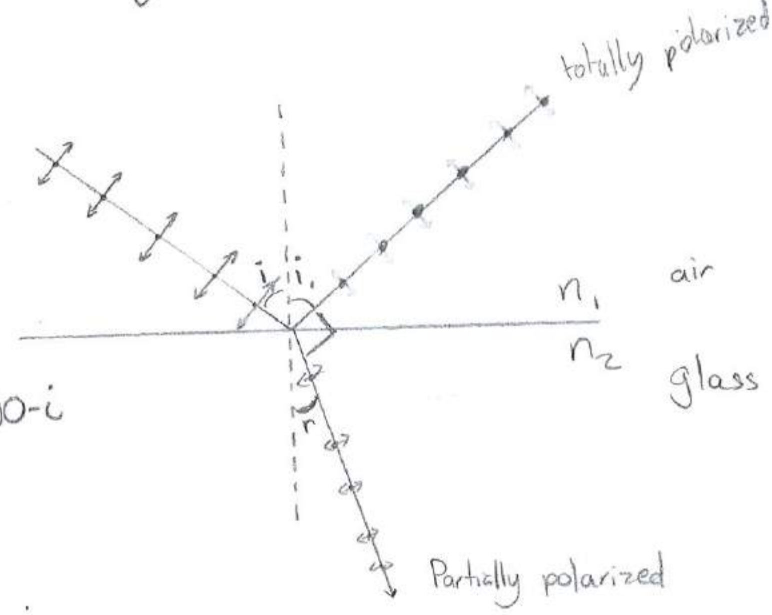
E - electric field
B - magnetic field

plane polarized EM Wave.

Plane of polarisation is vertical (electric field vector)



Brewster angle



$r = 90 - i$

i_B = Brewster angle of incidence = $\tan i_B = \frac{n_2}{n_1}$ ← denser medium / ← lighter medium

LCDs

- each pixel made of a tiny liquid crystal.
    - usually rotate plane polarization at 90°, but when current passes through them they don't.
    - if crystal plazed between 2 polarizers, when current goes throught light stops

Doppler effect

Moving Source

$$f' = f\left(\frac{v}{v \pm u_s}\right)$$

v = speed of sound

u = speed of source

+ = away (observer)

− = towards (obse

Moving observer

$$f' = f\left(\frac{v \pm u_o}{v}\right)$$

+ = towards (sourc

− = away (source

with light we employ approximation:

$$\Delta f = \frac{v}{c} f_0$$

Δf = chang in frequency

v = relative speed of the source & observer

c = speed of light in a vacuum

f_0 = original frequency

light moving away becomes red shifted

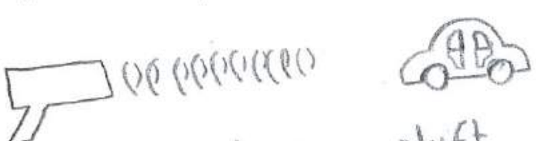
light moving toward becomes blue shifte

$$\Delta f = \frac{v}{c} f_0$$

$$\Delta \lambda = \frac{v}{c} \lambda_0$$

$v << c$

Speed Traps



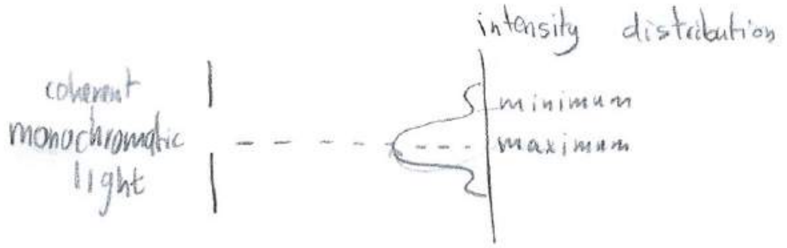
double doppler shift

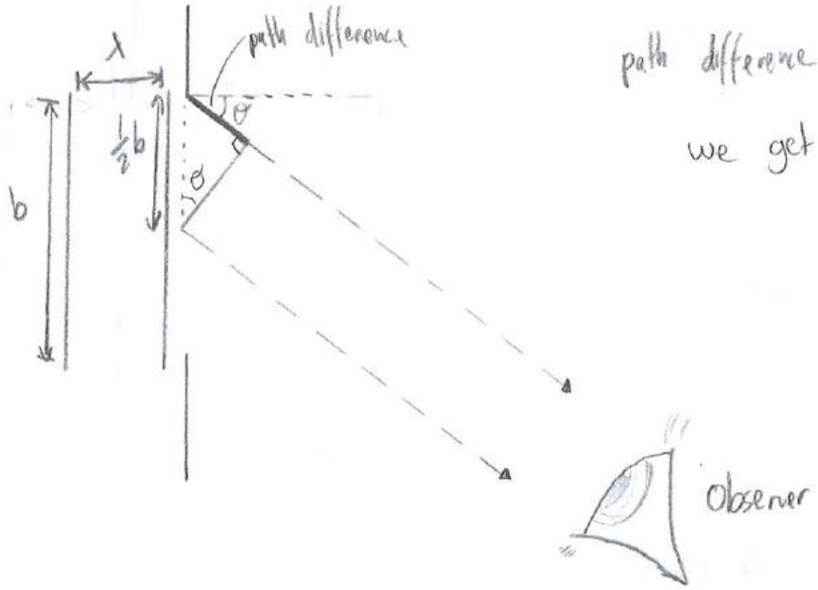
shift due to moving observer

shift due to moving source.

Single Slit Diffraction

short wave length do not deffract well.





path difference $= \frac{b}{2} \sin\theta$

we get deconstructive interference if $\lambda/2 =$ path diff

$$\frac{\lambda}{2} = \frac{b}{2}\sin\theta$$

$$\Rightarrow \frac{\lambda}{b} = \sin\theta$$

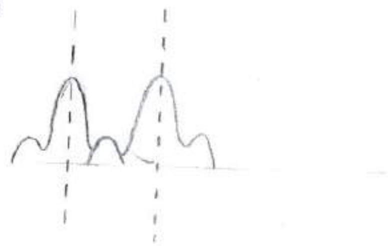
for small θ
$\sin\theta = \theta$

$$\frac{\lambda}{b} = \theta \quad \text{in rad}$$

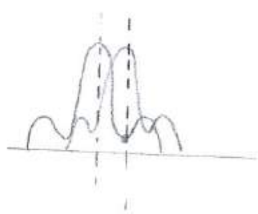
Resolution

Rayleigh criterion puts a limit for resolvability of 2 points

Resolved

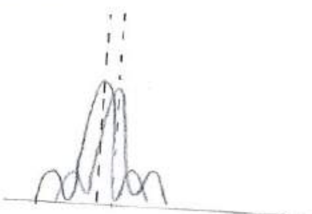


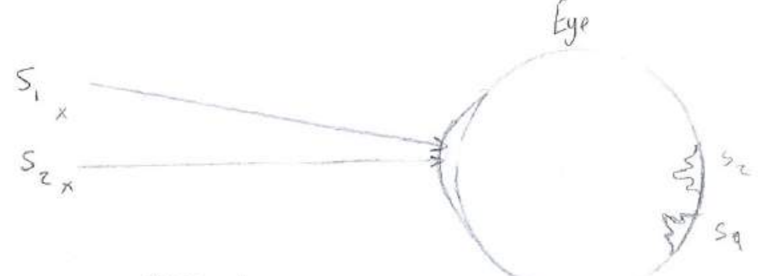
Just resolved



Two objects are just resolved if central maxima lies beyond the first minima of the other.

Not resolved





Diffractive pattern is circular so you introduce 1.22 constant $\theta = \frac{\lambda}{b} 1.22$

Newtons law of gravitation:

Every single <u>point mass</u> attracts every other point mass with a force proportional to the product of their masses and inversly proportional to their distance appart squared

Gravitational field strength

- force per <u>unit mass</u> experienced by a <u>small test mass</u> that is placed in the field.

$$g = \frac{F}{m}$$

Big test mass might affect gravitational field

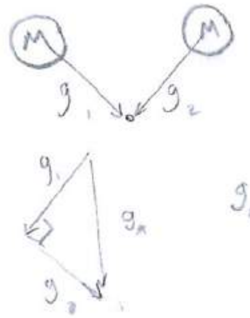
$$g = G\frac{m_1}{r^2}$$

Fields

- Field is a place in space where something is to be found.

gravitational field = place in spce where gravity is to be found.

when adding field strengths add them vectorically



$$g_R = \sqrt{g_1^2 + g_2^2}$$

Potential

PE = mgh ⇒ depends on mass (m) & position (gh)

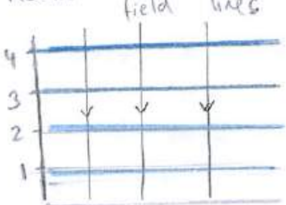
(gh) = where it is = gravitational potential (V)

$$PE = mgh \Rightarrow gh = \frac{PE}{m} = Jkg^{-1}$$

gravitational potential = potential energy per unit mass

↳ gravitational potential (where it is)

Equipotentials



equipotentials

field uniform ∴ equipotentials parallel & equally spaced

① moving along equipotential = no work done = moving perpendicular (cos 90) to field lines.

Work = $\Delta V \times m$ ← mass

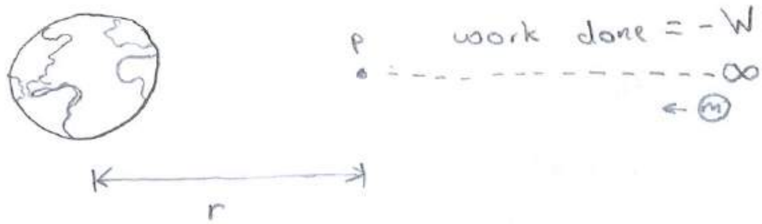
↑ change in potential $\frac{gh}{h} = g$

field strength = potential gradient = $\frac{\Delta V}{\Delta h}$ ← change in height

Gravitational potential at point P is defined as:

- work done per unit mass taking a small object from a position of zero potential to point P.

position of zero potential = ∞ distance

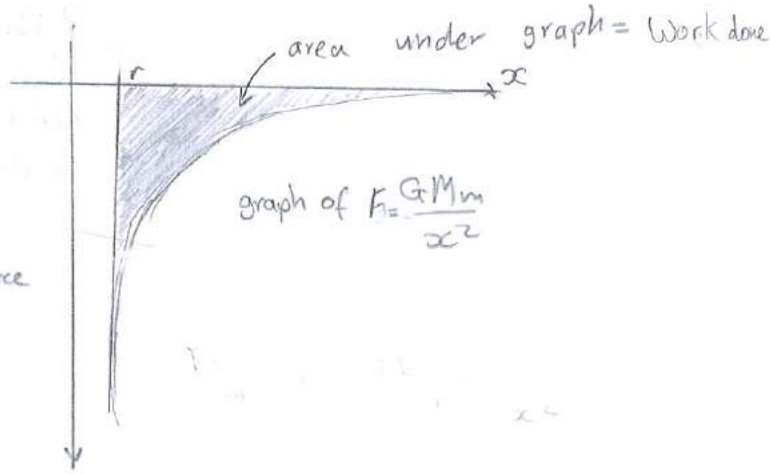


work done making the journey = -W $\quad V = \frac{-W}{m}$

$$W = \int_{\infty}^{r} \frac{GM}{x^2}\,dx =$$

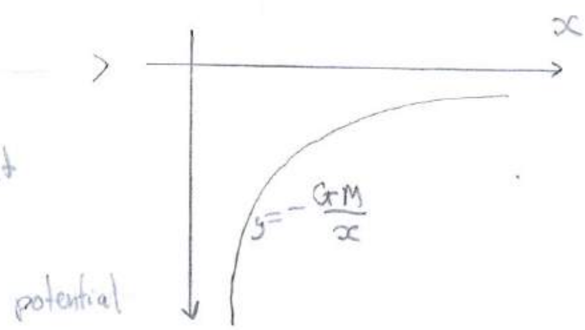
$$W = \left[\frac{GMm}{x}\right]_{\infty}^{r} = \frac{GMm}{r} \text{ force}$$

$$V = \frac{W}{m} = \frac{GM}{r}$$

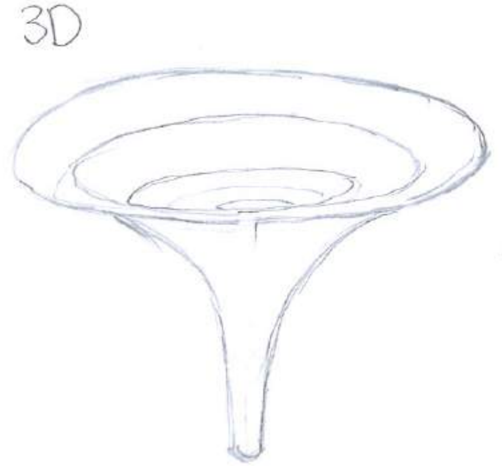


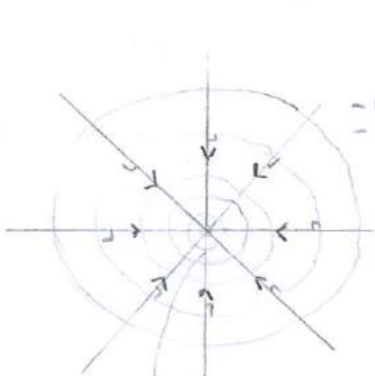
g = field strength $= -\frac{\Delta V}{\Delta x}$

$= -$ gradient



3D





- concentric circles
- strengthening field strength

field lines perpendicular to tangents of equipotentials

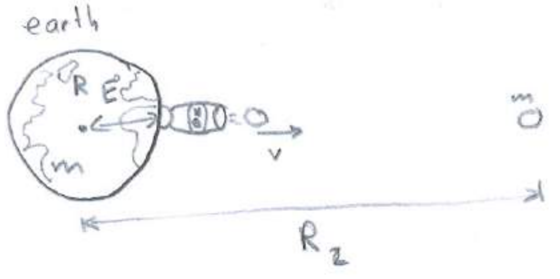
closer the lines, stronger the field

$\Rightarrow$ agrees with $g = -\frac{\Delta V}{\Delta x}$

- if drawn in 3d, lines would be spheres

equipotential line $\Rightarrow$ equipotential surface

Escape Velocity



as body is shot straight up, its KE decreases as it rises & PE increases, When it stops moving KE = 0 & PE = initioll KE (if no air resistance)

therefore $mgh = \frac{1}{2}mv^2$

$$= -\frac{GMm}{r}$$

loss of KE = gain of PE

$$\underset{\text{initial}}{\frac{1}{2}mv^2} - \underset{\text{final}}{0} = \underbrace{\left(-\underset{\text{final PE}}{\frac{GMm}{R_2}}\right) - \left(-\underset{\text{starting PE}}{\frac{GMm}{R_E}}\right)}_{\Delta PE}$$

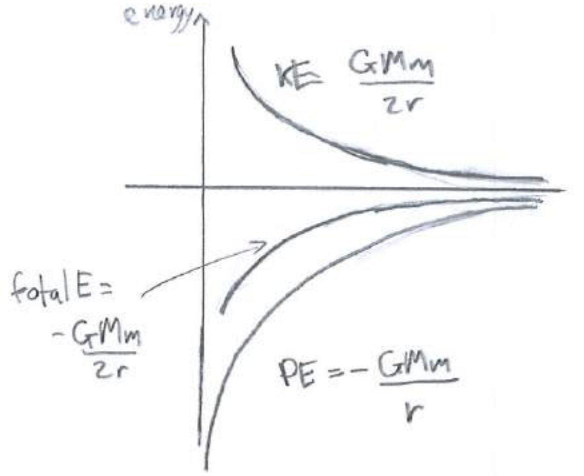
$\Delta KE = \Delta PE$

Escape velocity would occure when ball no longer fall back & escapes field lines, at this point $R_2 = \infty$

$$\frac{1}{2}mv^2 = -\underset{R_2 \to \infty \; = 0}{\frac{GMm}{\infty}} - -\frac{GMm}{R_E}$$

$$\therefore \quad \frac{1}{2}mv^2 = \frac{GMm}{R_E}$$

rearange to get $v = \sqrt{\frac{2GM}{R_E}}$



Orbital motion

For body to travel in circular path there must be an unbalaced force : centripetal force

centripetal force

$F = \frac{mv^2}{r}$ speed version $\Rightarrow$ becaus $F = ma$ $\quad a = \frac{v^2}{r}$ $\quad F = m\frac{v^2}{r}$

$F = m\omega^2 r$ angular speed version

centripetal force = acceleration in circular motion

$$m\omega^2 r = \frac{GmM}{r^2}$$

ω = angular speed , angle swept out (in rad) per unit time.

$$\omega = \frac{2\pi}{T}$$

$$m\left(\frac{2\pi}{T}\right)^2 r = \frac{GMm}{r^2}$$

$\hookrightarrow$ $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$ memorise

Keplers third Law

M = mass of star
G = constant
$4\pi^2$ = constant

$\Rightarrow \therefore \frac{T^2}{r^3}$ = constant in a solar system as M is constant

Energy of Orbiting Body

planets in orbit have KE due to movement & PE due to position

$$\text{Total E} = PE + KE = -\frac{GMm}{r} + \frac{GMm}{2r} = -\frac{GMm}{2r}$$

PE

KE

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

centripetal force

$$\frac{1}{2}mv^2 = -\frac{GMm}{2r} = kE$$

$$V = -\frac{GM}{r}$$

$$PE = Vm = -\frac{GmM}{r}$$

memorise

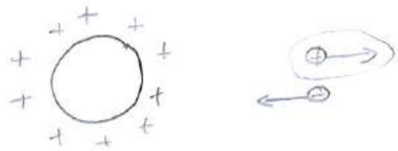
From this we learn that closer satelites desptite higher KE, hae a lower E_{total} than further satelites

Electric force

Conservation Of charge

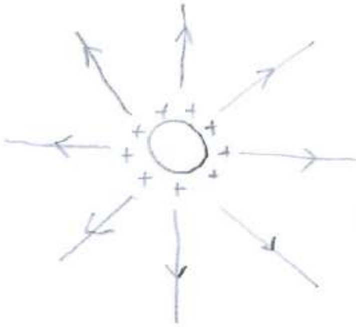
- we can take & add charge but we can't destroy it.

Electrical field: point in space where charged object experiences a force due to its charge

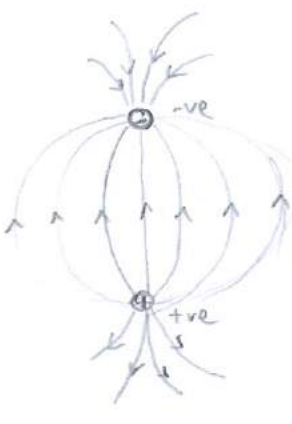


? 2 possible directions in the field

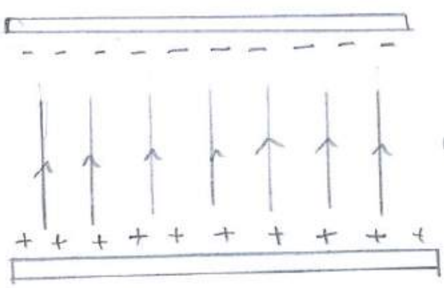
∴ therefore we consider direction as what would happen if +ve small charge was placed in the field.



Field lines close to a sphere charge



field lines due to dipole



uniform field

Coulomb's Law

- The force experienced by two point charges is directly proportional to the product of their charge & inversly proportional to the square of their separation
(similar to the Newton's Universal law of gravitation)

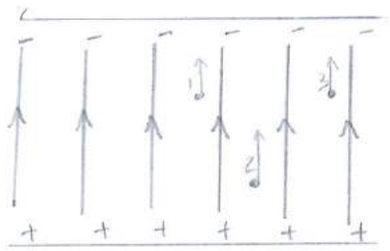
$$F = k\,\frac{Q_1 Q_2}{r^2}$$

Electric field strength (~~e~~ equivalent of gravitational field strength)

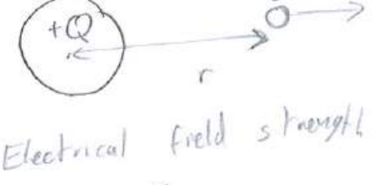
- the force experienced per ~~+~~ unit charge by a ~~+ve~~ test charge placed in the field.

$$E = \frac{F}{q}$$

↑ electrical field strength



1, 2, 3 all experience the same electrical field strength & Force



Electrical field strength felt by q

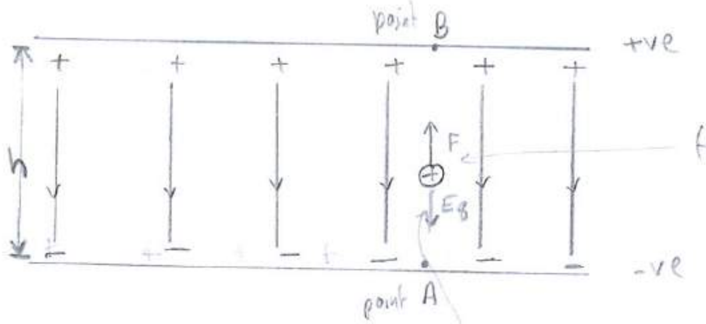
$$E = \frac{F}{q}$$

$$F = k\,\frac{Qq}{r^2}$$

$$\frac{F}{q} = k\,\frac{Qq}{r^2 q} = E = k\,\frac{Q}{r^2}$$

much like gravitational field strength, E has to be added up vectorially.

Electrical Potential



force we exert to move charge against electrical force & move ⊕ charge from A → B.

force experience by charge due to field.

as uniform field E is constant throughout

$$\frac{F}{q} = E \qquad Eq = F$$

Work done = PE given to chare = $F \times d = Eqh = E_p$

Definition

E_p, Electric Potential at a point is the <u>work per unit charge</u> needed to take the <u>small positive test charge</u> from a place of <u>zero potential</u> to the point.

Electric Potential (V_B) is scalar

V is in JC^{-1} $\quad V_B = Eh = V_B$

$$V_B = Eh = \frac{Eqh}{q} = \frac{E_p}{q} \qquad \therefore V_B = \frac{E_p}{q}$$

$$\Delta V = E \Delta h$$

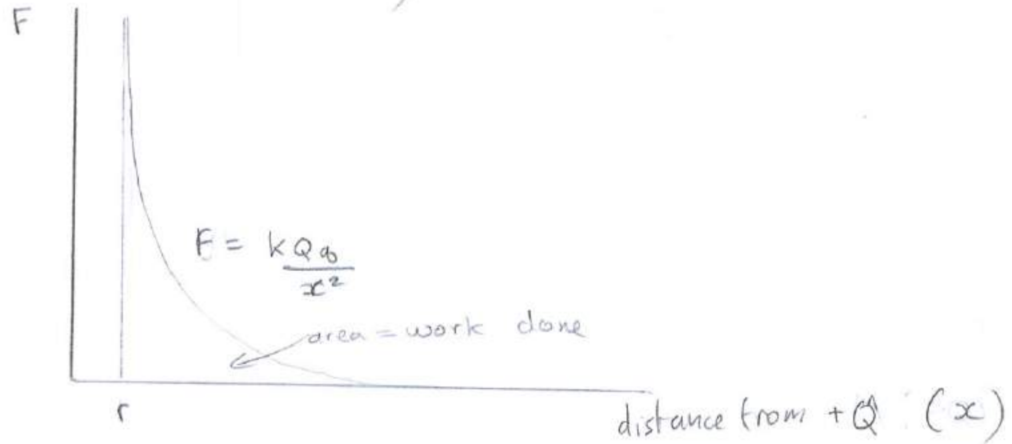
$$E = \frac{\Delta V}{\Delta h}$$ electrical field strength = rate of change of

eV, the electron volt:

1 eV, energy gained by an electon when accelerated through a p.d of 1 V.

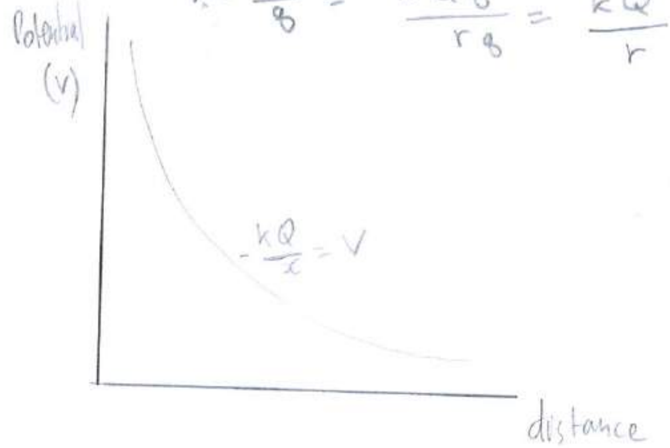
Potential in non uniform fields

Potential at P = work done per unit charge in bringing a small positive test charge from infinity to point P.



$$W = \int_r^\infty \frac{kQq}{x^2} = \left[\frac{kQq}{x}\right]_r^\infty = \frac{kQq}{r}$$

$$V = \frac{W}{q} = \frac{kQq}{rq} = \frac{kQ}{r}$$



$$\frac{dV}{dx} = -\frac{kQ}{x^2}$$

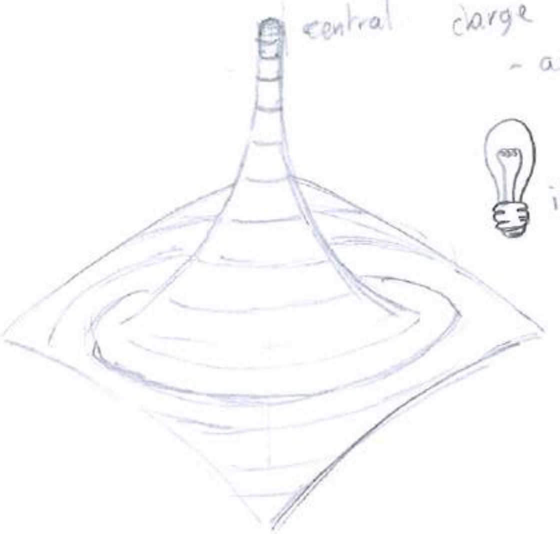
$$-\frac{kQ}{x^2} = -\frac{kQq}{x^2 q} = -\frac{F}{q} = -E$$

$$\therefore \frac{dV}{dx} = -E$$

$$E = \frac{\Delta V}{\Delta x}$$

electric field strength = - rate of change of electrical potential.

Equipotential Hills & wells



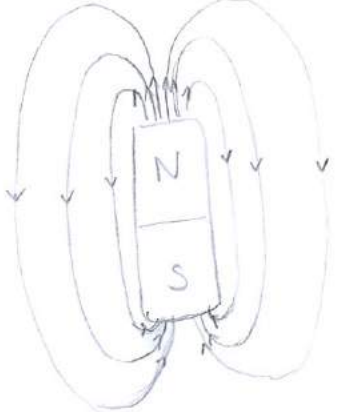
central charge is positive
- as we get closer V increases
hence the hill

if central charge is -ve, then a well forms

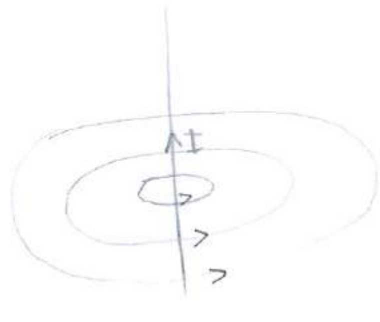
Magnetic Field

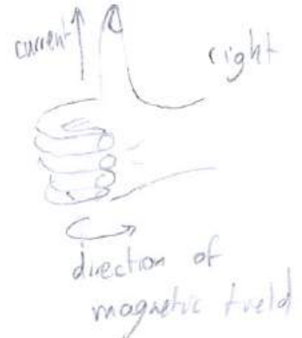
- A Magnetic Field, place in space where a small magnet experiences a turning force.

Field lines
- used to show where north would point if placed in the magnetic field.

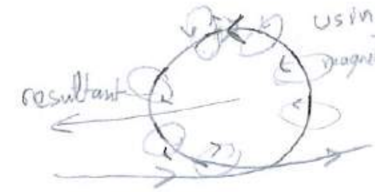


strength of field related to density of field lines. (however not same as in gravity or electricity)
Magnetic flux density (B)
⇒ deduce that magnetic flux strongest at poles.





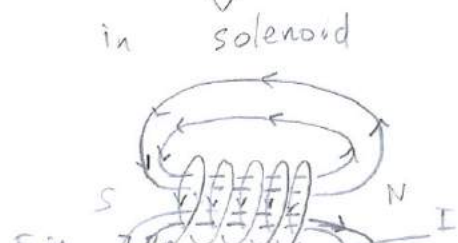
right hand rule



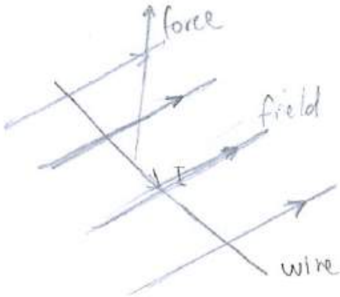
using right hand rule magnetic fields are as follow

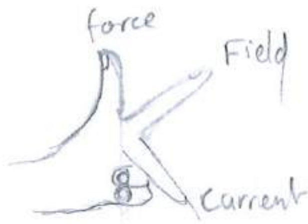
⇓

in solenoid



Force

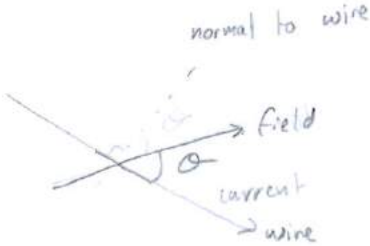


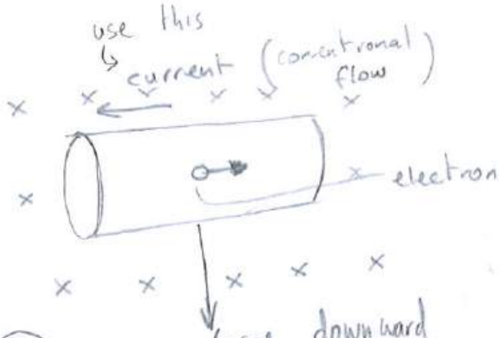


Force depends on

1. field strength - magnetic Flux density B
2. current
3. length of wire.

use $F = BIL \sin\theta$





x magnetic field

out of page ⇒

think of an arrow

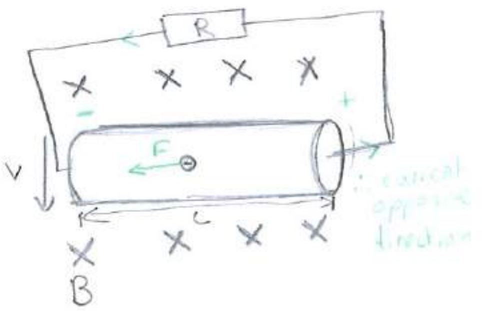
always take current as opposite of flow of e^-.

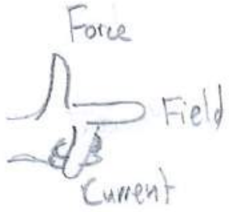
$F = Bqv$ is same as $F = BIL$

but for a single electron

Induction

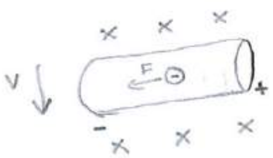
considering from electrons perspective





Current$_e$ = opposite to direction of electron
Field = into page
Force = FLHR gives the force on electron as being to the left.
current induced = opposite to the flow of the electron

Wire behaves as battery

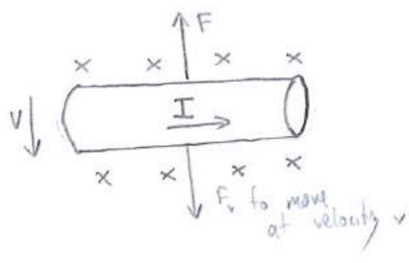


counter intuitive that e^- flows from + → −, however the wire cutting the B field is considered as a cell. hence charge flows opposite direction in it.

Induced emf (V)

- amount of mechanical energy converted to electrical energy per unit charge.

Conservation of energy



as the wire has a current through it and is placed in a magnetic field, it will experience an upwards force opposing the direction travelled in the B field. Hence to move wire through B field work must be done
Work done, ↑ electrical PE, then dissapated in circuit PE → Heat,

Calculating induced emf

max p.d occurs when

$$F_B = F_E$$

magetic Force ↑ ↑ electrical force

$F_B = Bqv \qquad q = e$

$= Bev$

$F_E = Ee$

$E = \frac{\Delta V}{\Delta x}$

uniform field $\therefore E = \frac{V}{L}$

$E = \frac{Ve}{L}$

$F_B = F_E = \frac{Ve}{L} = Bev$

$\mathcal{E} = V = BvL$

↑ area swept out

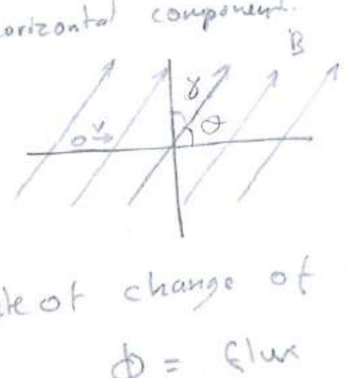
field strength

$\therefore \mathcal{E} =$ area swept out in field

if field & direction of motion not perpendicular consider area swept out $(v \cdot L)$ as only horizontal component.

$\mathcal{E} = B\sin\theta \times LV$

$= B\cos\gamma \times LV$ (horizontal component direct related)



Faraday's Law

The induced emf is equal to the rate of change of flux

$\mathcal{E} = \frac{d\Phi}{dt} \qquad \Phi = \text{flux}$

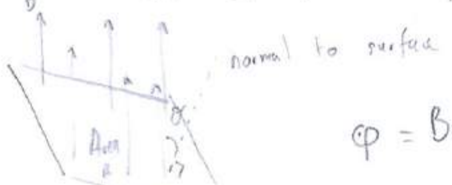
$\hookrightarrow \mathcal{E} = -N\frac{\Delta\phi}{\Delta t}$

Flux & Flux density

- Flux density $\propto$ number of field line per unit area (B)
- Flux $\propto$ number of field lines in a given area

unit of flux is tesla metre^2 (Tm^2) or webber (Wb)

unit of flux density is tesla (T) or webber per meter^2 (Wbm^{-2})



normal to surface

$\varphi = B\cos\theta \times A$

Lenz's Law

Any emf induced will act in such a way to oppose the change causing it (example of induced emf & FLHR)

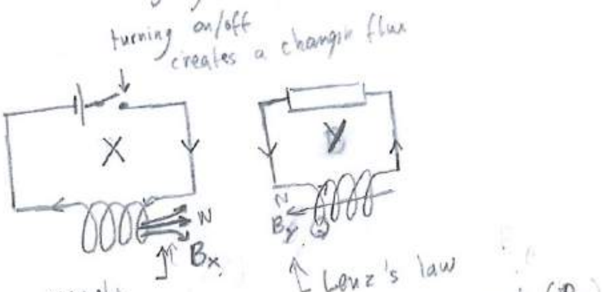
Lenz's law is direct consequence of conservation of energy. we need to do work to induce emf.

Magnet & coil

① Faraday' Law
- as magnet approaches coil, B field inside coil increase (changing ϕ). Induces emf...

② Lenz's Law
- induced emf will be such to oppose the change causing it (current in coil must create magnetic field such that it opposes the north of the magnet. Use right hand grip rule)

2 Coil in changing field



① Faraday's law
- changing flux in X, causes an emf to be induced in Y. (the changing ϕ causes, flux inclosed in Y to change, hence inducing ε)

② Lenz's Law
- emf induced must act to oppose the change causing it, Hence B_y must oppose B_x. Use RHGR to find current.

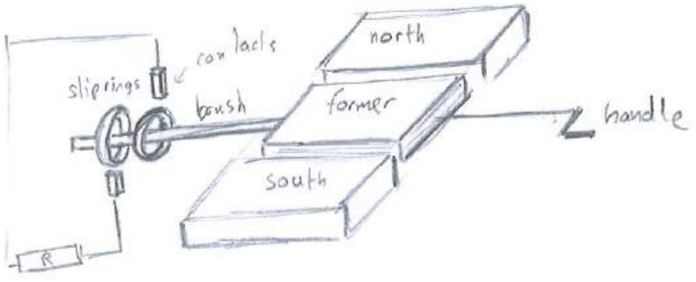
Applications of induction
- Braking: Friction pads replaced by electromagnets. (tesla)
- Induction cooking: hot plate has rotating magnet which induces emf into saucepan. this emf → heat.

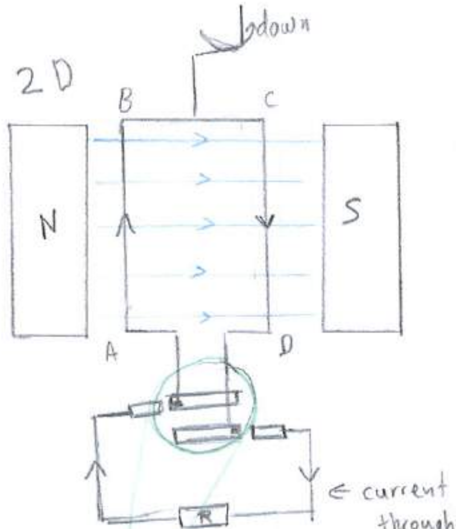
AC

Emf induction is primarily used to generate power (AC).

- Dynamo (coil rotates in magnetic field)
- Alternator (magnet rotates in coil.)

3D





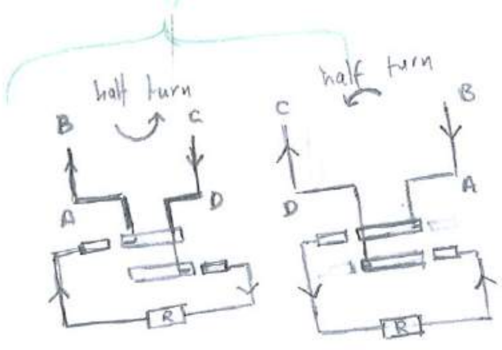
use FRHR to determine induced currents in wire.

motion AB ↑up
CD ↓down

Field same
(perpendicular to AB & CD) ⇒ emf induced
(parrallel to BC & AD) ⇒ no emf

current use FRHR

← current through circuit will change
(determined by position of brush (current in brush) on slip ring)



- current flips in circuit, as wires CD, AB have change in their currents direction (look at slip ring)
- the coil doesnt have a change in current (by simply looking)

size of induced emf

Using Faraday's Law → emf induced as flux changing.

Flux changing as flux enclosed in coil is changing due to angle coil makes with field (effective area changing)

Flux $\varphi = BA\cos\theta$

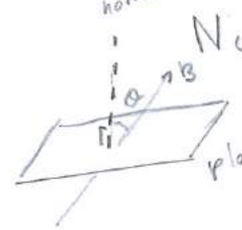
when more than 1 coil

$N\varphi = BAN\cos\theta$

assume θ to be angle when angular frequency is at time t.

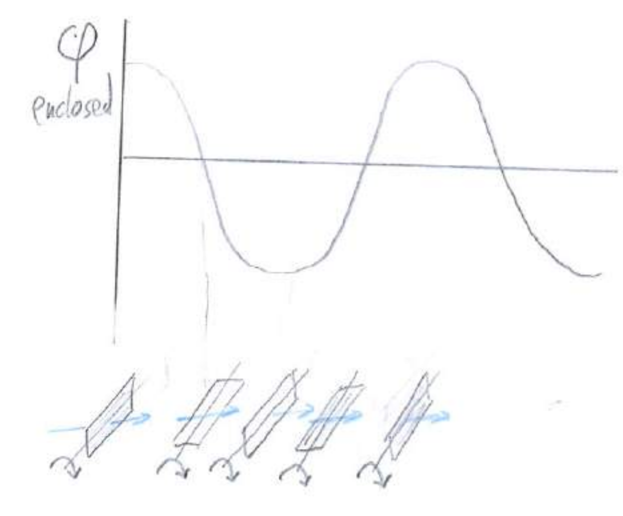
$\theta = \omega t$

$N\varphi = BAN\cos\omega t$

$\theta =$ 

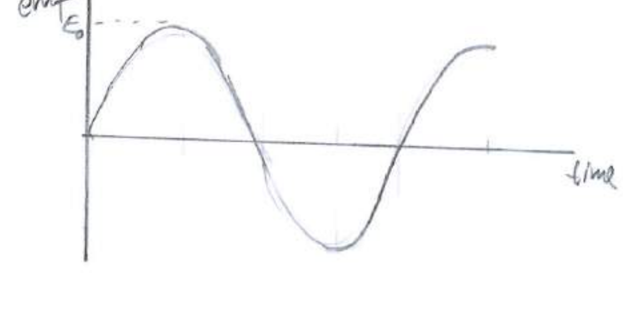
θ is angle between normal and field.

Graph φ vs time



Graph Emf vs time

Emf = rate of change $\frac{dy}{dx}$ & Lenz says it must <u>oppose</u> change causing it, hence emf $= -\frac{dy}{dx}$



$\varepsilon = BAN\omega\sin\omega t$

changing angular frequency ω changes amplitude & period.

Further Ac Currents

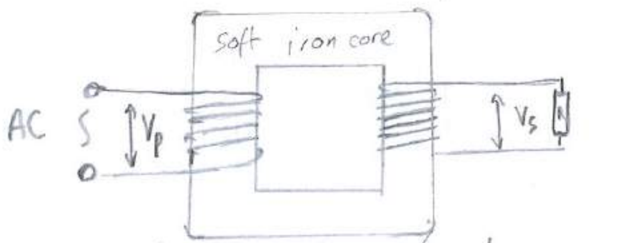
- Average $|\mathcal{E}| = \mathcal{E}_{rms} = \sqrt{\frac{E_0^2}{2}} = \frac{E_0}{\sqrt{2}}$

route mean square

- Average $I = I_{rms} = \frac{I_0}{\sqrt{2}}$

Transformer

Constituted of 2 wound cores around a soft iron core



1) primary coil cause a changing magnetic fiel due to ac current.
2) Soft iron core amplifies strength of field & itself becomes a temporary magnet
3) changing flux enclosed in secondary coil causes an induced emf

relation

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{I_s}{I_p}$$ flipel here

Idea that $V_p I_p = V_s I_s = P$

Power must remain constant.